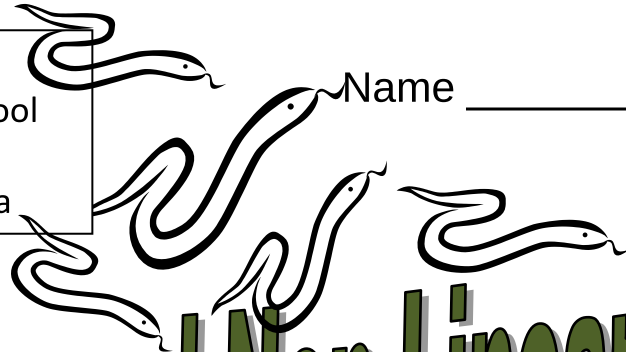
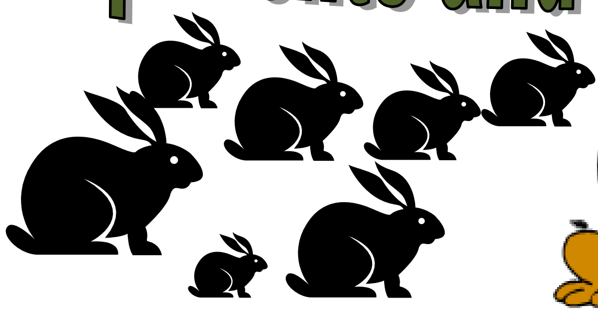
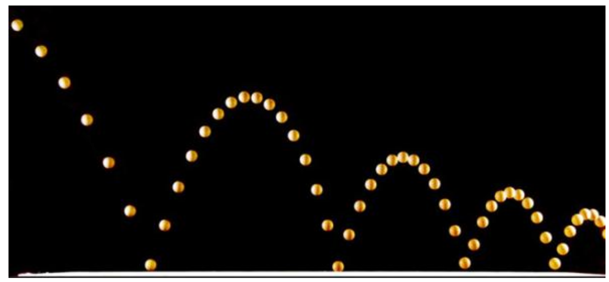
Unit 5
Beaumont Middle School
8th Grade, 2016-2017
Introduction to Algebra

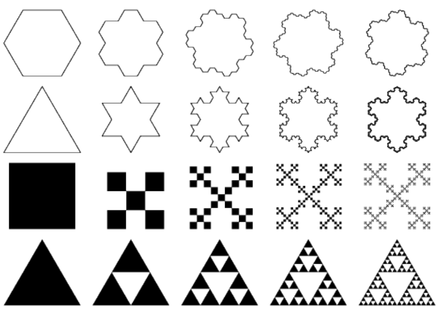
Name ______________________________

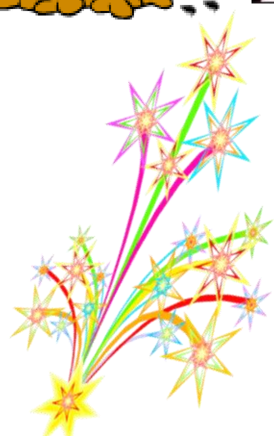


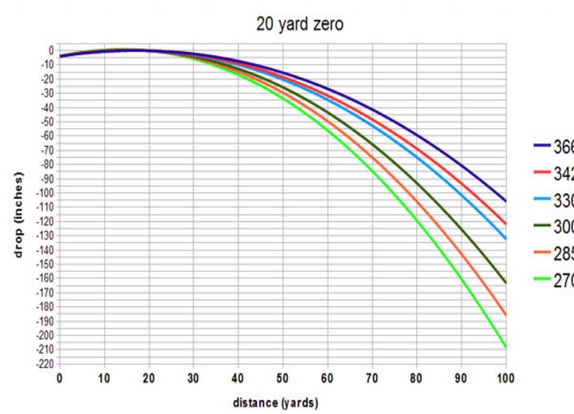
Exponents and Non-Linear Functions

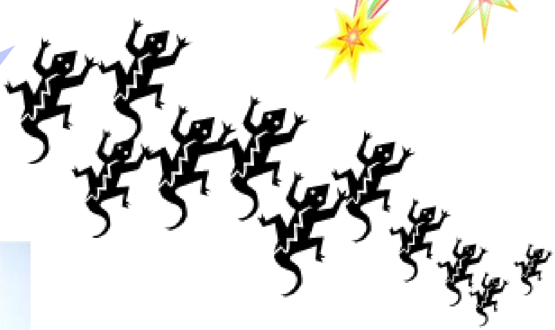


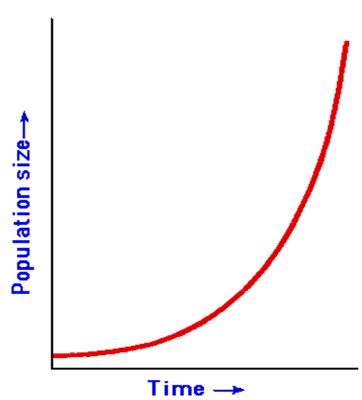












- I can simplify expressions involving integral exponents.
- Model an exponential relationship with a function table and graph.
- Model a quadratic relationship with a function table and graph.
- Convert numbers represented in scientific notation and standard form.
- Apply the properties of exponents to simplify expressions involving integral exponents.
- Perform operations with numbers expressed in scientific notation.

Objectives: I can simplify expressions involving integral exponents.

Negative Exponents

Complete the following tables. Leave all answers as integers or fractions.

2^5	32
2^4	16
2^3	
2^2	
2^1	
2^0	
2^{-1}	
2^{-2}	
2^{-3}	

3^5	243
3^4	81
3^3	
3^2	
3^1	
3^0	
3^{-1}	
3^{-2}	
3^{-3}	

10^5	100,000
10^4	10,000
10^3	
10^2	
10^1	
10^0	
10^{-1}	
10^{-2}	
10^{-3}	

Solving Negative Exponents

You already know that an exponent represents the number of times you have to multiply a number by itself. For example, 2^4 means 2*2*2*2. But what if your variable is being raised to a negative exponent? If you were given 2^{-4}, how would you multiply two by itself negative four times?

A negative exponent is equivalent to the inverse of the same number with a positive exponent. In other words:

$$x^{-7} = \frac{1}{x^7}$$

There is nothing special about solving a problem that includes negative exponentials. It's just an intermediate step that you may or may not want to complete to make things simpler. The best way to get comfortable with negative exponents is to work a few example problems that use them. Here are some samples:

$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

$-4^{-2} = -\frac{1}{4^2} = -\frac{1}{16}$

$6^0 = 1$

$4^{-3} = \frac{1}{4^3} = \frac{1}{64}$

$(-5)^{-2} = \frac{1}{(-5)^2} = \frac{1}{25}$

$-4^0 = -1$

Practice:

7^{-2} $(-1)^2$ -2^{-4} -6^2 8^0 10^{-3}

If $x = -2, y = -3, and\ z = 4$, evaluate the following.

$3x^3$ y^2z^2 $10x^{-2}$ z^{-2}

Homework: Leave all answers as fractions if applicable.

Simplify.

1. 9^2
2. -3^{-2}
3. 4^3
4. $(-2)^{-2}$
5. $(-6)^0$
6. -8^2
7. -1^4
8. -9^0
9. $(-3)^{-2}$
10. -5^{-2}
11. $(-4)^{-2}$
12. 7^{-2}
13. -10^5
14. $(-2)^3$
15. 6^{-2}
16. $(-3)^{-4}$

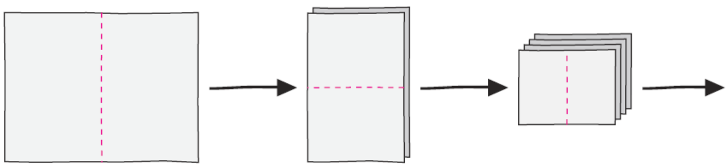
Evaluate if $x = -3, y = -2, and\ z = 5$.

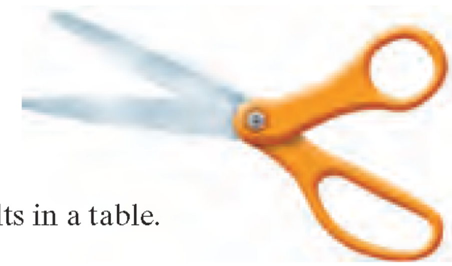
17. $3x^3$
18. y^2z^2
19. $10x^{-2}$
20. z^{-2}
21. $6y^{-3}$
22. $x^2 + 5x$
23. $-4y^3 + 2z$
24. $-x^{-2}$

Objectives: I can model an exponential relationship with a function table and graph.

Growing, Growing, Growing: Investigation 1

Chen, the secretary of the Student Government Association, is making ballots for tonight's meeting. He starts by cutting a sheet of paper in half. He then stacks the two pieces and cuts them in half. He stacks the resulting four pieces and cuts them in half. He repeats this process, creating smaller and smaller pieces of paper.





After each cut, Chen counts the ballots and records the results in a table.

Number of Cuts	Number of Ballots
0	1
1	2
2	4
3	
4	

Chen wants to predict the number of ballots after any number of cuts.

1. Predict how many ballots will result from 3 cuts. ________

2. Predict how many ballots will result from 4 cuts. ________

3. Predict how many ballots will result from 10 cuts. ________

4. Complete the 2nd column in the table to show the number of ballots after each of the cuts.

Number of Cuts (n)	Number of Ballots (b)	Calculations for Number of Ballots (b)	Shortcut Form for Number of Ballots using Exponents (b)
0	1	1	
1	2	[1] · 2	
2	4	[1 · 2] · 2	
3	8	[1 · 2 · 2] · 2	
4			
5			
6			
7			
8			
9			
10			

5. How did you find your entries in the table?____

6. <u>Fully complete the table above.</u>

What is the relationship between the number of ballots and the previous number of ballots?

____

7. What is the relationship between the number of cuts and the number of ballots? (In other words, how can you use the number of cuts to figure out the number of ballots?)

____

8. A rule (equation) to explain the relationship between of the number of cuts (*n*) and the number of ballots (*b*) is $\boldsymbol{b = 1 * 2^n}$. Use this rule (equation) to determine how many ballots Chen would have if he made 20 cuts?

Show your work. ____

9. Use the rule (equation) to determine how many ballots Chen would have if he made 30 cuts?

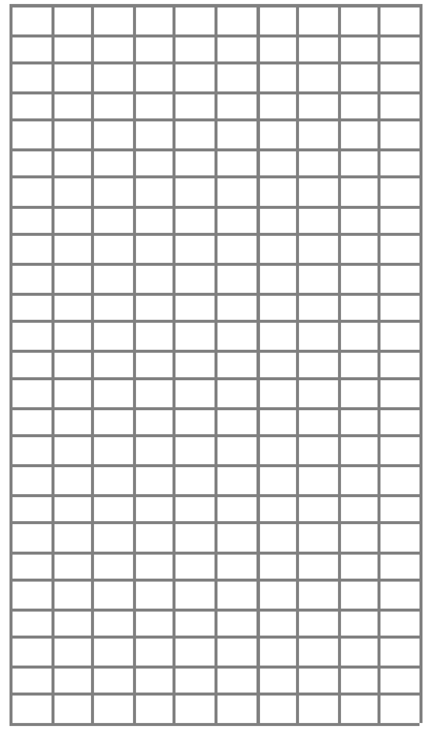
Show your work. ____

10. How many cuts would it take to make enough ballots for all 500 students in Chen's school? ________
Explain how you determined this answer. ____

____

12. Graph the relationship.

Use an interval of 1 on the x-axis and 50 on the y-axis.



When you found the number of ballots after 10, 20 and 30 cuts, you may have multiplied a long string of 2s. Instead of writing long product strings of the same factor, you can use the exponential form. For example, you can write $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ as 2^5, which is read as "2 to the fifth power."

In the expression 2^5, you get $2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$. We say that 32 is the standard form for 2^5.

13. Write each expression in exponential form.

a) $2 \cdot 2 \cdot 2$

b) $5 \cdot 5 \cdot 5 \cdot 5$

c) $1.5 \cdot 1.5 \cdot 1.5 \cdot 1.5 \cdot 1.5 \cdot 1.5 \cdot 1.5$

14. Write each expression in standard form.

a) 2^7

b) 3^3

c) 4.2^3

15. Most calculators have a $\boxed{\wedge}$ or $\boxed{y^x}$ key for evaluating exponents. Use your calculator to find the standard form for each expression.

a) 2^{15}

b) 3^{10}

c) 1.5^{20}

16. Explain how the meanings of 5^2, 2^5, $2 \cdot 5$ and $5 \cdot 2$ differ.

____

____

____

Write each expression in <u>exponential form</u>.

17. 2 * 2 * 2 * 2 ___________

18. 10 · 10 · 10 · 10 · 10 · 10 · 10 __________

19. 2.5 * 2.5 * 2.5 * 2.5 * 2.5 ___________

Write each expression in <u>standard form</u>.

20. 2^{10} 21. 10^2 22. 3^9

23. $(-5)^{-2}$ 24. -4^{-2} 25. $(-6)^{-2}$ 26. 10^{-2}

27. -2^5 28. $(-3)^3$ 29. 8^{-2} 30. $(-5)^{-4}$

Evaluate if $x = -1, y = -3, and\ z = 10$.

31. $3x^3$ 32. y^2z^2 33. $10x^{-2}$ 34. z^{-2}

Bacteria Growth

If you don't brush your teeth regularly, it won't take long for large colonies of bacteria to grow in your mouth. Suppose that a single bacterium lands on one of your teeth and starts reproducing by a factor of four every hour. (multiplies by four every hour)

b = the number of bacteria Equation: $\mathbf{b = 4^n}$

n = the number of hours

1. Complete the table and graph for the relationship.
(Use an interval of 1/2 on the x axis and 50 on the y axis.)

# of hours	# of bacteria

2. Use the equation to find how many bacteria will be in the new colony after 12 hours?

Show your work. ____

3. How many bacteria will be in the new colony after 13 hours? ____________________________
Explain how you can use your answer from #2 instead of using your equation.

____

4. After how many hours will there be at least 1 million bacteria in the colony ? ___________
(Use guess and check to find your answer.)

Objectives: I can model an exponential relationship with a function table and graph.

Growing, Growing, Growing: Investigation 2

1. Ghost Lake is a popular site for fisherman, campers, and boaters. In recent years, a certain water plant has been growing on the lake at an alarming rate. At present, 1,000 square feet are covered by the plant. The Dept. of Natural Resources estimates that the area is doubling every month.

A) An equation that represents the growth pattern of the plant on Ghost Lake is

$$a = 2^m * 1000$$

Explain what information the variables and numbers in the equation represent.

Numbers:____

Variables: ____

B) Make a table and a graph of the equation. (Use x-intervals of 1 and y-intervals of 1,000.)

C) The surface area of Ghost Lake is 25,000,000 square feet. How much of the lake's surface will be covered with the water plant by the end of the year? (Show your work.)

____

D) In how many months will the plant completely cover the surface of the lake? ______________

2. Loon Lake has a “killer plant” problem similar to Ghost Lake. Currently, 5,000 feet of the lake is covered with the plant. The area covered is growing by a factor of 1.5 each year.

a) Complete the table and graph to show the area covered by the plant for the next 5 years. (Use intervals of 1 for the x-axis and 1,000 for the y-axis.)

b) An equation that represents the growth pattern of the plant on Loon Lake is $y = 1.5^x * 5000$

1. Explain what information the variables and numbers in the equation represent.

Numbers:____

Variables: ____

c) Use the equation to determine how much of the lake's surface will be covered by the plant in 8 years. Show your work.

____

d) The surface area of the lake is approximately 200,000 square feet. How long will it take before the lake is completely covered? (Use your table to help determine the answer.)

____

3. In parts of the U.S., wolves are being reintroduced to wilderness areas where they have become extinct. Suppose 20 wolves were released in Northern Michigan, and the yearly growth factor for this population is expected to be 1.2.

a) Make a table showing the projected number of wolves at the end of each of the first 6 years.

b) An equation that models the growth pattern of the wolf population is $\boldsymbol{y = 1.2^x * 20.}$

Explain what information the variables and numbers in the equation represent.

Numbers:____

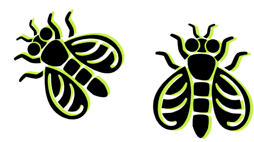
Variables: ____

c) Use this equation to determine how many wolves there would be in 8 years.

Show your work. ____

d) How long will it take for the new population to exceed 100? _________________________

4. Fruit flies are often used in genetic experiments because they reproduce at a phenomenal rate. In 12 days, a pair of *Drosophila* can mature and produce a new generation of fruit flies. The table below shows the number of fruit flies in three generations of a laboratory colony.



Generation	0	1	2	3
# of Flies	2	120	7200	432,000

a) What is the growth factor for this fruit-fly population? ____________

Show how you found the answer. ____

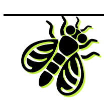
b) An equation that models this situation is $\boldsymbol{y = 60^x * 2.}$
Explain what information the variables and numbers in the equation represent.

Numbers:____

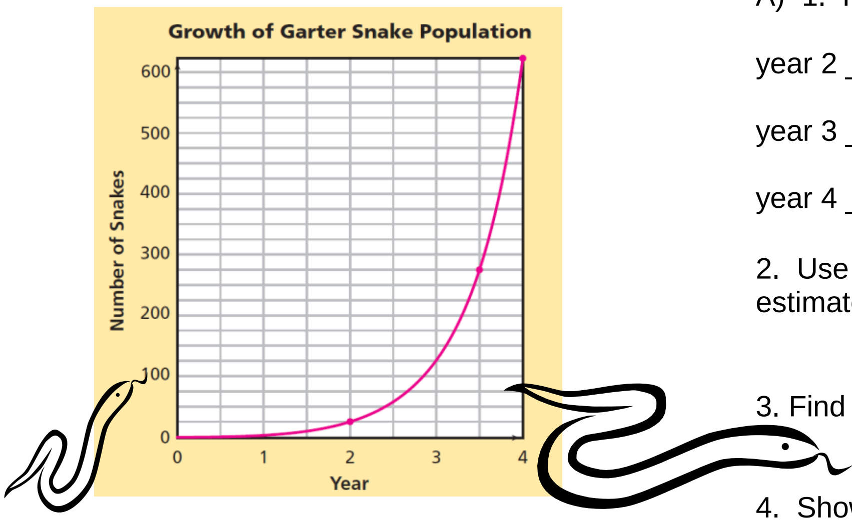
Variables: ____

c) If this pattern continues, how many fruit flies will there be in the fifth generation? Show your work.

____



5) The graph shows the growth of a garter snake population after it was introduced to a new area. The population is growing exponentially.



A) 1. Find the snake population for

year 2 ______________________________

year 3 ______________________________

year 4 ______________________________

2. Use the pattern in your answer from part 1 to estimate the population in year 1

____

3. Find the growth factor for the population.

____

4. Show how you can find the y-intercept for the graph. _____________

B) An equation that models this situation is $\boldsymbol{y = 5^x * 1}$.
Explain what information the variables and numbers in the equation represent.

Numbers:____

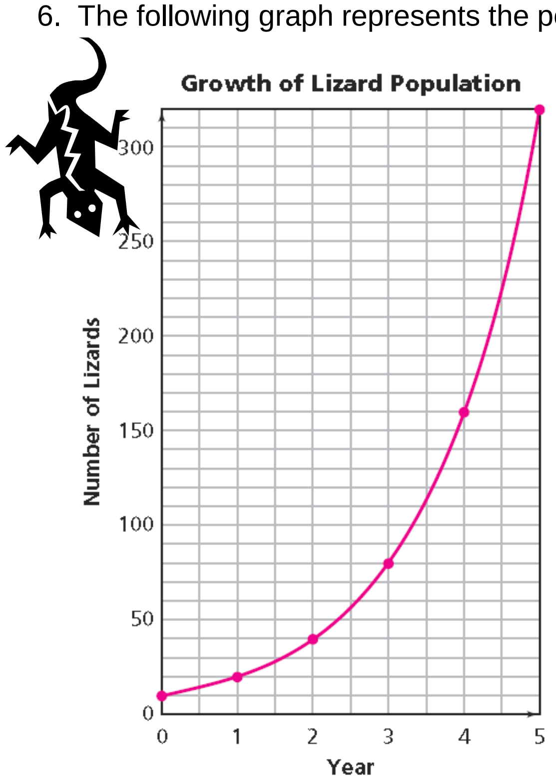
Variables: ____

C) Use the equation to determine how many snakes there would be in year 8. Show your work.

____

D) In what year is the population likely to reach 1,500? ____________________________

6. The following graph represents the population growth of a certain kind of lizard.



a) What information does the point (2,40) on the graph tell you?

____

b) What information does the point (1,20) on the graph tell you?

____

c) What is the growth factor of the population? (Show your work.)

____

d) What was the initial population? _______________

e) The equation relating time t in years and population is $p = 2^t * 10$. Use the equation to determine the population in 10 years.

____

f) When will the population exceed 100 lizards? ___________

g) When will the population exceed 600 lizards? ___________

Objectives: I can model an exponential relationship with a function table and graph.

Growing, Growing, Growing Investigation 3.1 and Scientific Notation

3.1 Reproducing Rabbits

1. In 1859, a small number of rabbits were introduced to Australia by English settlers. The rabbits had no natural predators in Austrailia, so they reproduced rapidly and became a serious problem, eating grasses intended for sheep and cattle.

If biologists had counted the rabbits in Australia in the years after they were introduced, they might have collected data like these:

A. The table shows the rabbit population growing exponentially.

Growth of Rabbit Population

Time (yr)	Population
0	100
1	180
2	325
3	583
4	1,050

1. What is the growth factor? _______ Show how you found your answer.______________

2. Assume the growth pattern continued. An equation for the rabbit population p for any year n after the rabbits were first introduced is $p = 1.8^n * 100$. Explain what the numbers in the equation represent.

____

3. How many rabbits will there be in 10 years? ________________

How many will there be after 25 years? _____________ After 50 years?________________

4. After how many years will the rabbit population exceed one million? _________________

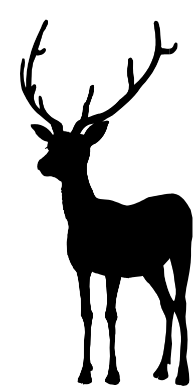
B. Suppose that during a different time period, the rabbit population could be predicted by the equation $p = 15\ (1.2^n)$, where p is the population in millions, and n is the number of years.

1. What is the growth factor? _____________ 2. What is the initial population? ___________

2. The table shows that the elk population in a state forest is growing exponentially.

Growth of Elk Population

Time (yr)	Population
0	30
1	57
2	108
3	206
4	391
5	743



a. What is the growth factor? ____________

Show how you got your answer. __________________

b. An equation you could use to predict the elk population p for any year n after the elk were first counted is $\boldsymbol{p = 30\ (1.9^n)}$, Suppose that this growth patterns continues.

How many elk will there be in 10 years? _______15 years? ________

d. After how many years will the elk population exceed one million? ____________________

Carp

As a biology experiment, Nicole is investigating how fast a particular carp population will grow under controlled conditions. She **starts her population with 5 carp**. After one month she counts 15 carp. If the population is reproducing exponentially, it will **multiply by 3** each month.

1. An equation for the relationship between the number of carp and the number of months.

Equation: $y = 5\,(3^x)$,

. Explain what information the variables and numbers in the equation represent.

Numbers:____

Variables: ____

2. Complete the table and graph for the relationship.
(Use an interval of 1/2 on the x axis and 50 on the y-axis.)

3. Use the equation to find how many carp Nicole will have in one year. ____________

4. How many carp will Nicole have after 13 months? __________

Explain how you can use your answer from #3 instead of using your equation.

____

5. If a relationship is linear, what does the graph look like?

____

6. If a relationship is linear, what pattern of change do you see in a table?

____

7. If a relationship is exponential, what does the graph look like?

____

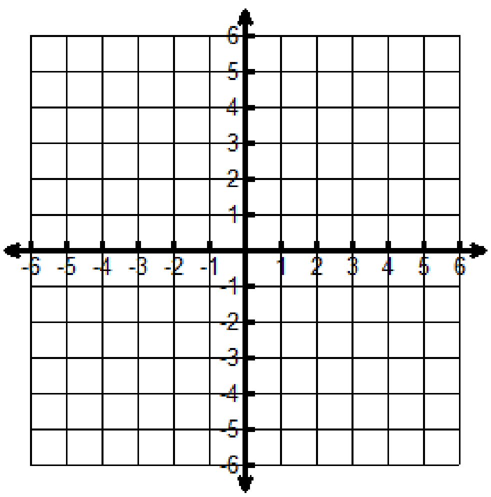
8. If a relationship is exponential, what pattern of change do you see in a table?

____

9. Graph each linear function.

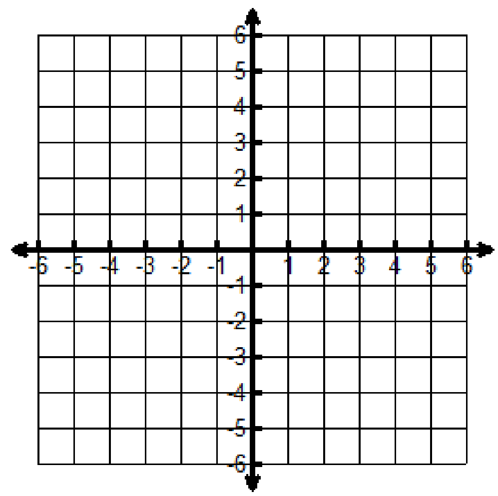
$y = \frac{-2}{3}x + 5$

$m =$ _____ $b =$ _______



$-4x + 10y = 20$

x and y-intercepts: _________ **and** __________



10. Simplify. $Use\ x = -10\ as\ needed.$

-3^2 $(-3)^2$ $-(-3)^2$ x^2 $-x^2$

7^{-2} $(-3)^{-2}$ -2^{-2} -6^{-2} 10^{-2}

Objectives: I can express numbers in standard form and scientific notation.

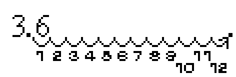
Scientific Notation and Standard Form (Decimal Notation) Notes

- By using exponents, we can reformat numbers. For very large or very small numbers, it is sometimes simpler to use "scientific notation" (so called, because scientists often deal with very large and very small numbers).
- The format for writing a number in scientific notation is fairly simple: (first digit of the number) followed by (the decimal point) and then (all the rest of the digits of the number), times (10 to an appropriate power). The conversion is fairly simple.

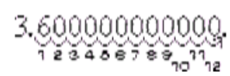
- **Write 124 in scientific notation.**
  This is not a very large number, but it will work nicely for an example. To convert this to scientific notation, I first write "1.24". This is not the same number, but (1.24)(100) = 124 is, and $100 = 10^2$. Then, in scientific notation, 124 is written as $\mathbf{1.24 \times 10^2}$.

- Actually, converting between "regular" notation and scientific notation is even simpler than I just showed; because all you really need to do is count decimal places.

- **Write in decimal notation: 3.6×10^{12}**
  Since the exponent on 10 is positive, I know they are looking for a LARGE number, so I'll need to move the decimal point to the right, in order to make the number LARGER. Since the exponent on 10 is "12", I'll need to move the decimal point twelve places over. First, I'll move the decimal point twelve places over. I make little loops when I count off the places, to keep track:



Then I fill in the loops with zeroes:



In other words, the number is **3,600,000,000,000**, or 3.6 trillion

- **Convert 93,000,000 to scientific notation.**
  This is a large number, so the exponent on 10 will be positive. The first "interesting" digit in this number is the leading 9, so that's where the decimal point will need to go. To get from where it is to right after the 9, the decimal point will need to move seven places to the left. Then the power on 10 will be a positive 7, and the answer is $\mathbf{9.3 \times 10^7}$

Scientific Notation and Standard Form (Decimal Notation) Practice

Write in standard form.

1) 4.0×10^3 ________________ **2)** 4.5×10^4 ________________ **3)** 6.5×10^5 ________________

4) 7.6×10^2 ________________ **5)** 8×10^3 ________________ **6)** 6.32×10^7 ________________

Write each number in scientific notation.

7) 465,000,000________________ **8) 98,000,000,000**________________ **9) 373,000** ________________

10) 697,000,000,000______________ **11) 54,000,000** ________________ **12) 24,340,000**________________

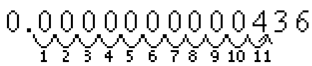
Use your calculator to evaluate the following. Write the answer in scientific notation and standard form.

	Scientific notation (3 significant digits)	Standard form
13) 7^{12}	____________________	_____________________
14) 12^{15}	____________________	_____________________
15) 4^{24}	____________________	_____________________
16) 18^9	____________________	_____________________

Scientific Notation and Standard Form (Decimal Notation) Notes

- By using exponents, we can reformat numbers. For very large or very **small** numbers, it is sometimes simpler to use "scientific notation" (so called, because scientists often deal with very large and very small numbers).
- The format for writing a number in scientific notation is fairly simple: (first digit of the number) followed by (the decimal point) and then (all the rest of the digits of the number), times (10 to an appropriate power). The conversion is fairly simple.

- **Write 0.000 000 000 043 6 in scientific notation.**
  In scientific notation, the number part (as opposed to the ten-to-a-power part) will be "4.36". So I will count how many places the decimal point has to move to get from where it is now to where it needs to be:

  Then the power on 10 has to be -11: "eleven", because that's how many places the decimal point needs to be moved, and "negative", because I'm dealing with a SMALL number. So, in scientific notation, the number is written as $\mathbf{4.36 \times 10^{-11}}$

- **Convert 4.2×10^{-7} to decimal notation.**
  Since the exponent on 10 is negative, I am looking for a small number. Since the exponent is a seven, I will be moving the decimal point seven places. Since I need to move the point to get a *small* number, I'll be moving it to the left. The answer is **0.000 000 42**

- **Convert 0.000 000 005 78 to scientific notation.**
  This is a small number, so the exponent on 10 will be negative. The first "interesting" digit in this number is the 5, so that's where the decimal point will need to go. To get from where it is to right after the 5, the decimal point will need to move nine places to the right. Then the power on 10 will be a negative 9, and the answer is $\mathbf{5.78 \times 10^{-9}}$

Just remember: However many spaces you moved the decimal, that's the power on 10. If you have a small number (smaller than 1, in absolute value), then the power is negative; if it's a large number (bigger than 1, in absolute value), then the exponent is positive.

Warning: A negative on an exponent and a negative on a number mean two very different things! For instance:

$$-0.00036 = -3.6 \times 10^{-4}$$
$$0.00036 = 3.6 \times 10^{-4}$$
$$36{,}000 = 3.6 \times 10^{4}$$
$$-36{,}000 = -3.6 \times 10^{4}$$

Don't confuse these!

Part 1) Write in standard form.

1) 4.82×10^{-5} ________________ 2) 2.6×10^{-7} ________________ 3) 1.79×10^{-4} ________________

4) -5.28×10^{5} ________________ 5) 7×10^{8} ________________ 6) -6.12×10^{-6} ________________

Write each number in scientific notation.

7) 0.000 000 000 52 ____________ 8) 0.000 000 041 ______________ 9) 0.000 000 398 ______________

10) 578,000,000______________ 11) 38,000,000,000 ___________ 12) 219,000 __________________

Use your calculator to evaluate the following. Write the answer in scientific notation and standard form.

	Scientific notation **(3 significant digits)**	Standard form
13) 5^{-11}	____________	____________
14) 9^{-12}	____________	____________
15) 8^{-15}	____________	____________
16) 5^{20}	____________	____________
17) 9^{11}	____________	____________
18) 10^{12}	____________	____________

Part 3) Scientific Notation

Write in standard form.

1) $1.91 \text{ x } 10^{-5}$ ____________ 2) $3.1 \text{ x } 10^{7}$ ____________ 3) $8.46 \text{ x } 10^{8}$ ____________

4) $9.182 \text{ x } 10^{5}$ ____________ 5) $7.2 \text{ x } 10^{-8}$ ____________ 6) $1.97 \text{ x } 10^{-6}$ ____________

Write each number in scientific notation.

7) 0.000 000 027 ____________ 8) 79,000,000,000____________ 9) 0.000 000 398 ____________

10) 123,100,000____________ 11) 0.000 000 05 ____________ 12) 0.000 000 018____________

Use your calculator to evaluate the following. Write the answer in scientific notation and standard form.

	Scientific notation	Standard form
13) 7^{-11}	____________	____________
14) 8^{12}	____________	____________
15) 5^{-15}	____________	____________
16) 6^{-10}	____________	____________
17) 7^{14}	____________	____________
18) 12^{-8}	____________	____________

Objectives: I can apply the properties of exponents to simplify expressions involving integral exponents.

Properties of Exponents

Explore…look for patterns

Complete to see why the rules for exponents work.

1. $4^5 \cdot 4^2 = (\_\_)\ (\_\_)\ (\_\_)\ (\_\_)(\_\_) \cdot (\_\_)(\_\_) = 4^{\_\_}$

2. $8^3 \cdot 8 = (\_\_)\ (\_\_)\ (\_\_) \cdot (\_\_) = 8^{\_\_}$

3. $4^5 \div 4^2 = \dfrac{4^5}{4^2} = \dfrac{\cancel{4} \cdot \cancel{4} \cdot 4 \cdot 4 \cdot 4}{\cancel{4} \cdot \cancel{4}} = 4^{\_\_}$

4. $8^3 \div 8 = \dfrac{8^3}{8} = \dfrac{\cancel{8} \cdot 8 \cdot 8}{\cancel{8}} = 8^{\_\_}$

5. $\dfrac{6^3}{6^3} = 6^{3-3} = 6^{\_}$ Also, $\dfrac{6^3}{6^3} = \dfrac{\overset{1}{\cancel{6}} \cdot \overset{1}{\cancel{6}} \cdot \overset{1}{\cancel{6}}}{\underset{1}{\cancel{6}} \cdot \underset{1}{\cancel{6}} \cdot \underset{1}{\cancel{6}}} = \_\_.$ So, $6^0 = \_\_.$

Complete to write each product or quotient as one power.

6. $12^3 \cdot 12^2 = 12^{3+2} = 12^{\_\_}$

7. $9^4 \cdot 9^3 = 9^{\_\_\_\_} = 9^{\_\_}$

8. $\dfrac{7^6}{7^2} = 7^{6-2} = 7^{\_\_}$

9. $\dfrac{12^6}{12^4} = 12^{\_\_\_\_} = 12^{\_\_}$

Write each product or quotient as one power.

10. $10^4 \cdot 10^6 = \_\_\_\_\_\_$

11. $5^5 \cdot 5 = \_\_\_\_\_\_$

12. $4^5 \cdot 4 \cdot 4^3 = \_\_\_\_\_\_$

13. $\dfrac{15^6}{15^2} = \_\_\_\_\_\_$

14. $\dfrac{9^5}{9} = \_\_\_\_\_\_$

15. $\dfrac{2^{10}}{2^2} = \_\_\_\_\_\_$

Properties of Exponents

To multiply powers with the same base, keep the base and add exponents.

$x^a \cdot x^b = x^{a+b}$

$4^5 \cdot 4^2 = 4^{5+2} = 4^7$

$8^3 \cdot 8 = 8^{3+1} = 8^4$

To divide powers with the same base, keep the base and subtract exponents.

$x^a \div x^b = x^{a-b}$

$4^5 \div 4^2 = 4^{5-2} = 4^3$

$8^3 \div 8 = 8^{3-1} = 8^2$

Any nonzero number raised to the zero power equals 1.

$x^0 = 1$ with $x \neq 0$

$17^0 = 1$

Multiplying and Dividing Monomials

Find each product or quotient. Express using exponents.

1. $2^3 \cdot 2^5$

2. $10^2 \cdot 10^7$

3. $1^4 \cdot 1$

4. $6^3 \cdot 6^3$

5. $(-3)^2(-3)^3$

6. $(-9)^2(-9)^2$

7. $a^2 \cdot a^3$

8. $n^8 \cdot n^3$

9. $(p^4)(p^4)$

10. $(z^6)(z^7)$

11. $(6b^3)(3b^4)$

12. $(-v)^3(-v)^7$

13. $11a^2 \cdot 3a^6$

14. $10t^2 \cdot 4t^{10}$

15. $(8c^2)(9c)$

16. $(4f^8)(5f^6)$

17. $\frac{5^{10}}{5^2}$

18. $\frac{10^6}{10^2}$

19. $\frac{7^9}{7^6}$

20. $\frac{12^8}{12^3}$

21. $\frac{100^9}{100^8}$

22. $\frac{(-2)^3}{-2}$

23. $\frac{r^8}{r^7}$

24. $\frac{z^{10}}{z^8}$

25. $\frac{q^8}{q^4}$

26. $\frac{g^{12}}{g^8}$

27. $\frac{(-y)^7}{(-y)^2}$

28. $\frac{(-z)^{12}}{(-z)^5}$

29. the product of two squared and two to the sixth power

30. the quotient of ten to the seventh power and ten cubed

31. the product of y squared and y cubed

32. the quotient of a to the twentieth power and a to the tenth power

Objectives: I can model a quadratic relationship with a function table and graph.

Quadratic Functions

Another type of non-linear function is a quadratic function. The shape is called a parabola which looks kind of like a U. We frequently see this type of function when gravity affects an object jumping or being thrown. We will explore a couple of situations that can be represented with a quadratic function.

No matter how hard you kick or throw a ball into the air, gravity always returns it to Earth. In this problem, you will see how the height of a football changes over time.

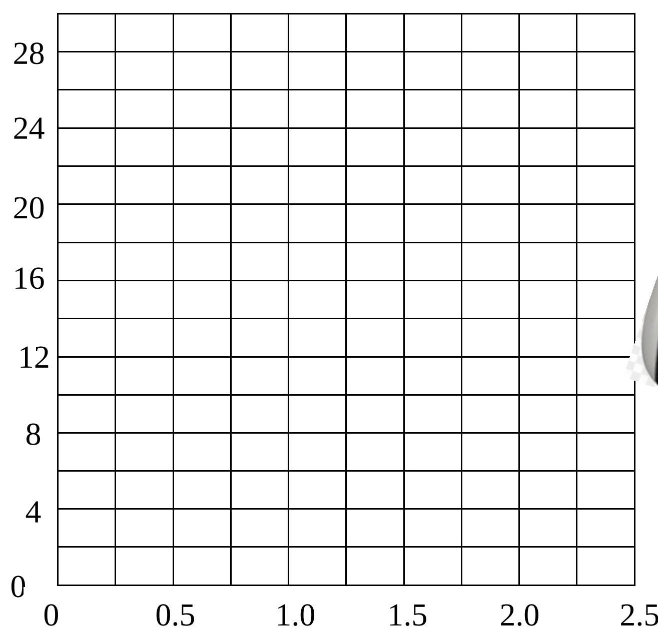
Suppose you filmed a ball as it is thrown straight up as high as possible. If you studied the films frame by frame, you would find that the time, t, in seconds and the H, height, in feet are related by an equation similar to this:

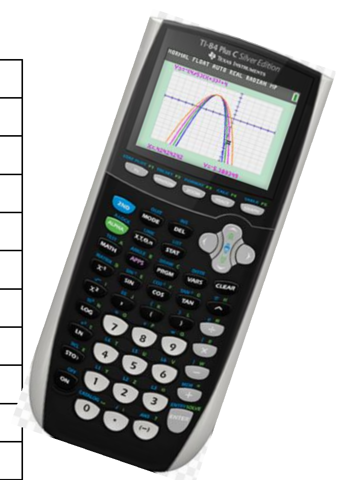
t = time (seconds) Football Equation: $\mathbf{H = -16t^2 + 40t + 4}$

H = the height (feet)

1. Complete the table and graph for the relationship. (Round the height to the nearest tenth of a foot.)

Time (seconds)	Height (feet)
0.0	
0.25	
0.5	
0.75	
1.0	
1.25	
1.5	
1.75	
2.0	
2.25	
2.5	





2. Describe the pattern of change for the ball in the height over time, and explain how the pattern is reflected in the table or the graph.

____

____

Linear or Nonlinear

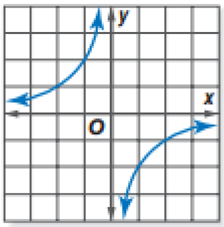
Graph: If the relationship is represented as a graph, then a line is linear!

Circle the correct response for each.

1) linear

or

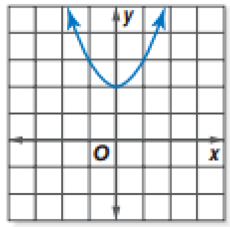
non-linear



2) linear

or

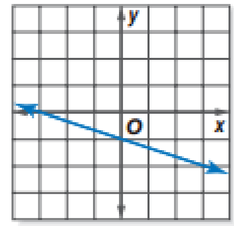
non-linear



3) linear

or

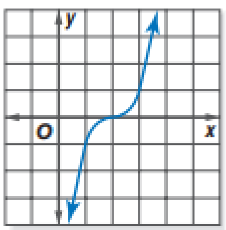
non-linear



4) linear

or

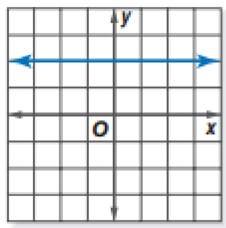
non-linear



5) linear

or

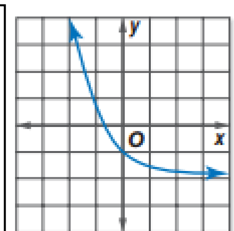
non-linear



6) linear

or

non-linear



Equation: If the relationship is represented as an equation, then try to write the equation in slope-intercept form or $y = mx + b$.

To be linear….
NO exponent with the variables
NO square root with the variables
NO absolute value bars around the variables
NO variable in the denominator

Circle the correct response for each.

1) linear

or

non-linear

$y = x^3 - 1$

2) linear

or

non-linear

$y = 4x^2 + 9$

3) linear

or

non-linear

$y = 0.6x$

4) linear

or

non-linear

$y = \frac{3x}{2}$

5) linear

or

non-linear

$y = \frac{4}{x}$

6) linear

or

non-linear

$y = \frac{8}{x} + 5$

Table: If the relationship is represented as a table, then the rate of change must be the same through the table. If the rate of change is constant this is called the slope in a linear relationship.

	x	y	
	2	50	
+2			−15
	4	35	
+2			−15
	6	20	
+2			−15
	8	5	

As x increases by 2, y decreases by 15 each time. The rate of change is constant, so this function is linear.

	x	y	
	1	1	
+3			+15
	4	16	
+3			+33
	7	49	
+3			+51
	10	100	

As x increases by 3, y increases by a greater amount each time. The rate of change is not constant, so this function is nonlinear.

Circle the correct response for each.

1) linear *or* non-linear

x	0	5	10	15
y	20	16	12	8

2) linear *or* non-linear

x	0	2	4	6
y	0	2	8	18

3) linear *or* non-linear

x	0	3	6	9
y	−3	9	21	33

4) linear *or* non-linear

Length (in.)	1	4	8	10
Width (in.)	64	16	8	6.4

Practice. **Determine whether each table, graph, or equation represents a *linear* or *nonlinear* function. Explain.**

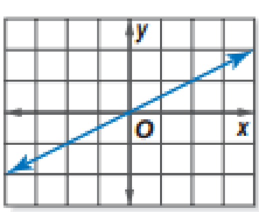
1.

x	0	1	2	3
y	1	3	6	10

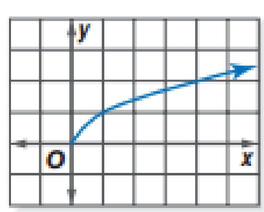
2.

x	0	3	6	9
y	−3	9	21	33

3.



4.



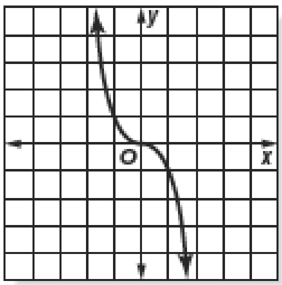
5. $y = \frac{x}{3}$

6. $y = 2x^2$

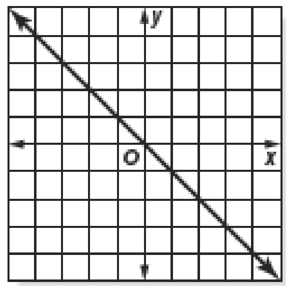
On your own:

Determine whether each graph, equation, or table represents a *linear* or *nonlinear* function. Explain.

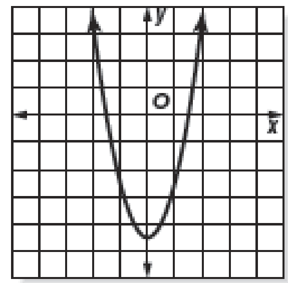
1.



2.



3.



4. $y = \frac{x}{2} + 1$

5. $y = \frac{2}{x} + 10$

6. $y = 8x$

7. $y = 6$

8. $2x - y = 5$

9. $y = x^2 + 4$

10. $y + 4x^2 - 1 = 0$

11. $2y - 8x + 11 = 0$

12. $y = \sqrt{3x} - 2$

13.

x	y
1	8
2	5
3	2
4	−1

14.

x	y
6	1
12	3
18	6
24	10

15.

x	y
20	−4
15	−2
10	0
5	2

Homework is continued on the next page.

Review Scientific Notation

Express each number in standard form.

1. 3.754×10^5
2. 8.34×10^6
3. 1.5×10^{-4}
4. 2.68×10^{-3}

Express each number in scientific notation.

5. 4,510,000
6. 0.00673
7. 0.000092
8. 11,620,000

9. **PHYSICAL SCIENCE** Light travels 300,000 kilometers per second. Write this number in scientific notation.

10. **TECHNOLOGY** The distance between tracks on a CD and DVD are shown in the table. Which disc has the greater distance between tracks?

Disc	Distance (mm)
CD	1.6×10^{-3}
DVD	7.4×10^{-4}

Replace each ● with <, >, or = to make a true sentence.

11. 2.3×10^5 ● 1.7×10^5
12. 0.012 ● 1.4×10^{-1}

Review Properties of Exponents

Simplify. Express using exponents.

1. $4^5 \cdot 4^3$
2. $5^2 \cdot 5^5$
3. $r^7 \cdot r^3$
4. $n^2 \cdot n^9$
5. $-2a(3a^4)$
6. $5^2x^2y^4 \cdot 5^3xy^3$
7. $\frac{7^6}{7}$
8. $\frac{2^{13}}{2^9}$

9 $\frac{y^8}{y^5}$

10. $\frac{z^2}{z}$
11. $\frac{9c^7}{3c^2}$
12. $\frac{24k^9}{6k^6}$

1)

Measuring Jumps

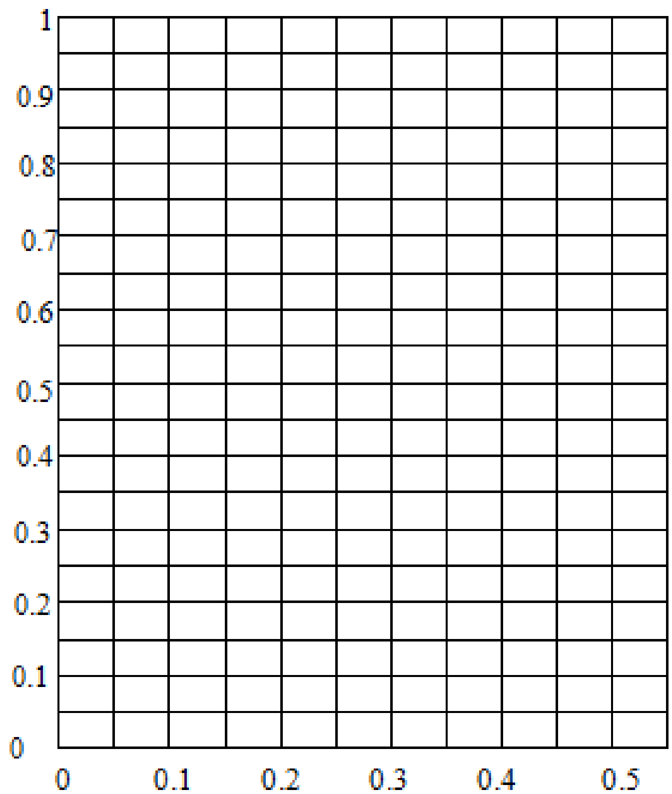
Suppose you filmed a flea as it jumped straight up as high as possible. If you studied the films frame by frame, you would find that the time, t, in seconds and the H, height, in feet are related by an equation similar to this:

t = time (seconds)
H = the height (feet)

Flea Equation: $H = -16t^2 + 8t$

a) Complete the table and graph for the relationship. (Round the height to the nearest hundredth of a foot.)

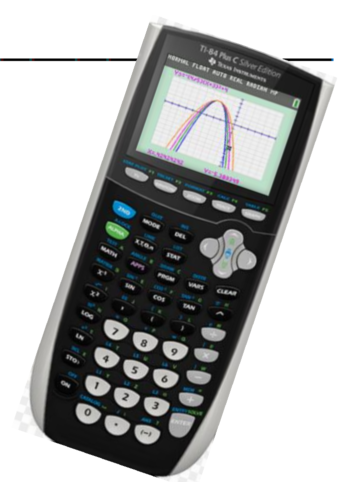
Time (seconds)	Height (feet)
0.0	
0.05	
0.1	
0.15	
0.2	
0.25	
0.3	
0.35	
0.4	
0.45	
0.5	



b) Describe the pattern of change for the flea in the height over time, and explain how the pattern is reflected in the table and the graph.

____

____



2) Square Numbers

a) Determine the number of smallest squares in each figure to complete the table.

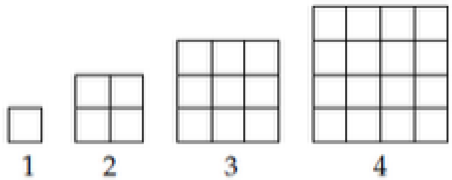


Figure Number						
# of Squares						

b) An equation that we can use to represent this relationship is $y = x^2$. What do the variables represent?

x: ____

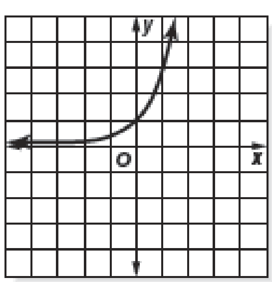
y: ____

c) Use the equation to determine the number of squares in the 15th figure. Show your work.

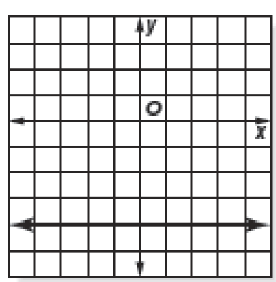
____

Determine whether each graph, equation, or table represents a *linear* or *nonlinear* function. Explain.

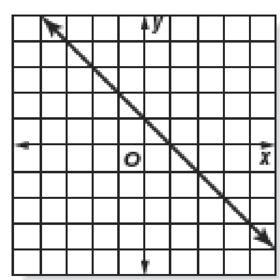
1.



2.



3.



4. $5x - y = 15$

5. $3y + 12x^2 = 0$

6. $5y - x + 3 = 0$

7. $y = 6\sqrt{x} + 10$

8. $y = \frac{8}{x}$

9. $y = -x^2 + 2$

10.

x	y
1	1.0
2	0.8
3	0.6
4	0.4

11.

x	y
44	0
48	2.5
52	5.0
56	7.5

12.

x	y
3	1
6	−2
9	−5
12	−14

Review

Scientific Notation

Express each number in standard form.

1. 1.5×10^3

2. 4.01×10^4

3. 6.78×10^2

4. 5.925×10^6

5. 7.0×10^8

6. 9.99×10^7

7. 3.0005×10^5

8. 2.54×10^5

9. 1.75×10^4

10. 1.2×10^{-6}

11. 7.0×10^{-1}

12. 6.3×10^{-3}

13. 5.83×10^{-2}

14. 8.075×10^{-4}

15. 1.1×10^{-5}

16. 7.3458×10^7

Express each number in scientific notation.

17. 1,000,000

18. 17,400

19. 500

20. 803,000

21. 0.00027

22. 5300

23. 18

24. 0.125

25. 17,000,000,000

26. 0.01

27. 21,800

28. 2,450,000

29. 0.0054

30. 0.000099

31. 8,888,800

32. 0.00912

Choose the greater number in each pair.

33. 8.8×10^3, 9.1×10^{-4}

34. 5.01×10^2, 5.02×10^{-1}

35. 6.4×10^3, 900

36. 1.9×10^{-2}, 0.02

37. 2.2×10^{-3}, 2.1×10^2

38. 8.4×10^2, 839

Order each set of numbers from least to greatest.

39. 3.6×10^4; 5.8×10^{-3}; 2.1×10^6; 3.5×10^5

Objectives: I can simplify expressions involving numbers expressed in scientific notation.

Operations with Numbers Expressed in Scientific Notation

Multiplication

When numbers in scientific notation are multiplied, only the number is multiplied. The exponents are added.

$$(2.00 \times 10^3)(4.00 \times 10^4) = (2.00)(4.00) \times 10^{3+4}$$

$$= 8.00 \times 10^7$$

Division

When numbers in scientific notation are divided, only the number is divided. The exponents are subtracted.

$$\frac{9.60 \times 10^7}{1.60 \times 10^4} = \frac{9.60}{1.60} \times 10^{7-4}$$

$$= 6.00 \times 10^3$$

Perform the following operations and express the answers in scientific notation.

a. $(4.3 \times 10^8) \times (2.0 \times 10^6)$

b. $(6.0 \times 10^3) \times (1.5 \times 10^{-2})$

c. $(1.5 \times 10^{-2}) \times (8.0 \times 10^{-1})$

d. $\dfrac{7.8 \times 10^3}{1.2 \times 10^4}$

e. $\dfrac{8.1 \times 10^{-2}}{9.0 \times 10^2}$

f. $\dfrac{6.48 \times 10^5}{(2.4 \times 10^4)(1.8 \times 10^{-2})}$

Operations with Scientific Notation

Simplify. Write each answer in scientific notation. Round to three significant digits if needed.

1) $\dfrac{4.6 \times 10^{-3}}{3 \times 10^{-6}}$

2) $(9 \times 10^{-5})(7.07 \times 10^{-3})$

3) $\dfrac{5 \times 10^{4}}{3 \times 10^{3}}$

4) $\dfrac{9.9 \times 10^{-5}}{1.3 \times 10^{-6}}$

5) $(5.8 \times 10^{-6})(6 \times 10^{-3})$

6) $(3.24 \times 10^{-4})(4.21 \times 10^{-6})$

7) $(5 \times 10^{6})(2.6 \times 10^{2})$

8) $\dfrac{6.74 \times 10^{-5}}{9 \times 10^{-3}}$

9) $(3.6 \times 10^{3})(5.1 \times 10^{4})$

10) $(9.1 \times 10^{5})(3.2 \times 10^{3})$

11) $\dfrac{9.7 \times 10^{3}}{5 \times 10^{4}}$

12) $\dfrac{5.04 \times 10^{4}}{2.2 \times 10^{2}}$

Operations with Scientific Notation

Simplify. Write each answer in scientific notation. Round to three significant digits if needed.

1) $(7 \times 10^{4})(5.6 \times 10^{6})$

2) $(6 \times 10^{-2})(4.71 \times 10^{-3})$

3) $(8.1 \times 10^{3})(5.7 \times 10^{6})$

4) $\dfrac{7.85 \times 10^{-2}}{8 \times 10^{-4}}$

5) $\dfrac{6.58 \times 10^{-2}}{4.2 \times 10^{-4}}$

6) $\dfrac{3 \times 10^{-4}}{2.4 \times 10^{-6}}$

7) $(8.3 \times 10^{3})(5 \times 10^{5})$

8) $\dfrac{7.4 \times 10^{4}}{6.3 \times 10^{6}}$

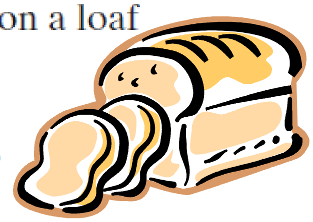
9) $(4 \times 10^{-6})(3 \times 10^{-5})$

10) $(8.15 \times 10^{-2})(9.2 \times 10^{-3})$

11) $\dfrac{4.6 \times 10^{6}}{2.6 \times 10^{3}}$

12) $\dfrac{6.17 \times 10^{3}}{5.63 \times 10^{5}}$

Review for Unit Test



1. Mold can spread rapidly. For example, the area covered by mold on a loaf of bread left out in warm weather grows exponentially.

Students at Magnolia Middle School conducted an experiment. They set out a shallow pan containing a mixture of chicken bouillon (BOOL yahn), gelatin, and water. Each day, the students recorded the area of the mold in square millimeters.

The students wrote the exponential equation $m = 50(3^d)$ to model the growth of the mold. In this equation, m is the area of the mold in square millimeters after d days.

A. What is the area of the mold at the start of the experiment? ________________

B. What is the growth factor? ________________

C. What is the area of the mold after 5 days? ________________

D. On which day will the area of the mold reach 6,400 mm^2? ________________

E. An exponential equation can be written in the form $y = a(b^x)$, where a and b are constant values.

   1. What value does b have in the mold equation? What does this value represent? ____

   2. What value does a have in the mold equation? What does this value represent? ____

2. A population of starlings is growing exponentially. An equation that represents the growth is $y = 3^x * 12$.

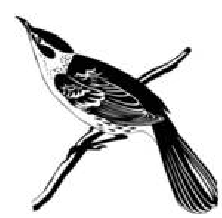
   a. Explain what information the variables and numbers in the equation represent.

   Numbers:____

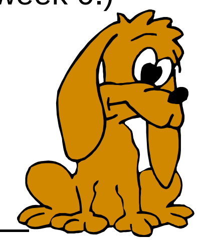
   Variables: ____

c. Make a table showing the population of starlings for the first 5 months.

d. Graph the population growth for 4 months. (Use interval of 1 on the x axis, 100 on the y axis.)



3. Fido did not have fleas when his owners took him to the kennel. The number of fleas on Fido after he returned from the kennel grew according to the equation $f = 8(3^n)$, where f is the number of fleas and n is the number of weeks since he returned from the kennel. (Fido left the kennel at week 0.)

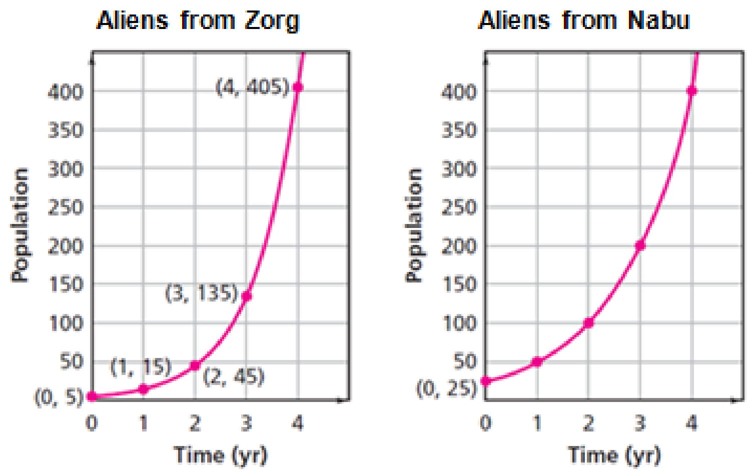


a. How many fleas did Fido pick up at the kennel? ____________

b. What is the growth factor for the number of fleas? ____________

c. How many fleas will Fido have after 10 weeks if he is not treated? _________________

4.



a) Find the growth factors for the two species. Which species is growing faster?

____

b) What are the y-intercepts for graphs of the Aliens from Zorg and Nabu? Explain what these y-intercepts tell you about the populations.

____

____

5. **Multiple Choice** Choose the answer that best approximates 3^{20} in scientific notation.

A. 3.5×10^{-9} **B.** 8×10^3 **C.** 3×10^9 **D.** 3.5×10^9

6. **Multiple Choice** Choose the answer that is closest to 2.575×10^6.

F. 2^{18} **G.** 12^6 **H.** 6^{12} **J.** 11^9

Scientific Notation and Standard Form (Decimal Notation) Practice

Write in standard form.

1) $6 * 10^7$ ________________ **2)** $4.52 * 10^6$ ________________ **3)** $3.521 * 10^5$ ________________

4) $8.5 * 10^{-6}$ ________________ **5)** $5 * 10^{-7}$ ________________ **6)** $3.26 * 10^{-5}$ ________________

Write each number in scientific notation.

7) 825,000,000________________ **8) 42,000,000,000**________________ **9) 846,000,000**________________

10) 0.000 015 6________________ **11) 0.000 000 008**________________ **12) 0.000 000 045** ________________

Use your calculator to evaluate the following. Write the answer in scientific notation and standard form. Round to three significant digits.

	Scientific notation	**Standard form**
13) 8^{15}	____________________	____________________
14) 14^{16}	____________________	____________________
15) 6^{-20}	____________________	____________________
16) 11^{-8}	____________________	____________________

Simplify. Leave all answers as fractions if applicable. (No calculator to be used.)

17. 9^2 **18.** -3^{-2} **19.** 4^3 **20.** $(-2)^{-2}$

21. $(-6)^0$ **22.** -8^2 **23.** -1^4 **24.** -9^0

Write each product or quotient as one power. No calculator to be used.

1. $12^3 \cdot 12^2$ **2.** $9^{-4} \cdot 9^{12}$ **3.** $\frac{7^8}{7^2}$ **4.** $\frac{12^{-8}}{12^7}$ **5.** $\frac{2^8}{2^{-7}}$

6. $8^4 \cdot 8^{-10}$ **7.** $x^4 \cdot x^2$ **8.** $\frac{a^{10}}{a^3}$ **9.** $5^{-4} \cdot 5^9$ **10.** $\frac{6^{-5}}{6^{-2}}$

11. $2^{-3} \cdot 2^{-2}$ **12.** $\frac{4^{-8}}{4^2}$ **13.** $x^{-3} \cdot x^{10}$ **14.** $\frac{1^8}{1^{-5}}$ **15.** $\frac{x^8}{x^{-7}}$

Simplify and express each number in scientific notation.

16. $(1.5\ x\ 10^{-3}) * (4.2\ x\ 10^{15})$ Scientific Notation: ____________________________

17. $\frac{2.8\ x\ 10^{-10}}{2\ x\ 10^{3}}$ Scientific Notation: ____________________________

18. $(2.1\ x\ 10^{-3}) * (8\ x\ 10^{-5})$ Scientific Notation: ____________________________

19. $\frac{8.8\ x\ 10^{10}}{0.2\ x\ 10^{3}}$ Scientific Notation: ____________________________

20. $(3.2\ x\ 10^{-4}) * (8\ x\ 10^{16})$ Scientific Notation: ____________________________

21. $\frac{4.5\ x\ 10^{-8}}{1.8\ x\ 10^{5}}$ Scientific Notation: ____________________________

22. $(6.8\ x\ 10^{8}) * (5.7\ x\ 10^{5})$ Scientific Notation: ____________________________