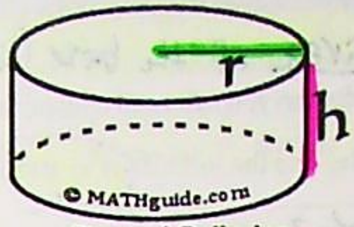
Volume of Cylinders Explained

The process for understanding and calculating the volume of cylinders is identical to that of **prisms**, even though cylinders are curved.

$V = Bh$

Here is a general **cylinder**.

The base is a circle.

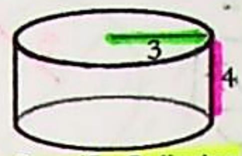


General Cylinder

Did you notice?

Just like with rectangular prisms, every cross section of a cylinder is the same as the bases. With cylinders the cross sections are all congruent: circle

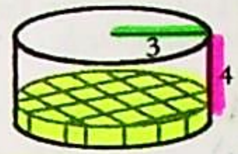
Let's start with a specific cylinder with **radius 3 units** and **height 4 units.**



Specific Cylinder

We will **fill the bottom** of the cylinder with unit cubes. This means the bottom of the prism will act as our **surface** and will be **covered** with as many unit cubes as possible without stacking them on top of each other yet.

This is what it would look like.



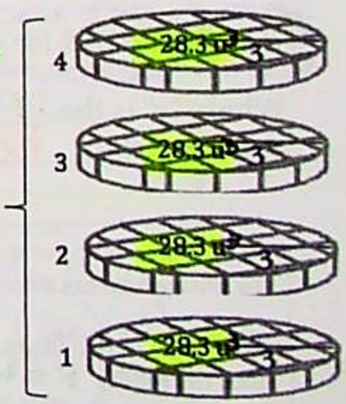
The diagram above is strange looking because we are trying to stack cubes within a curved space. Some cubes have to be shaved so as to allow them to fit inside. Also, the cubes do not yet represent the total volume. It only represents a partial volume, but we need to count these cubes to arrive at the total volume.

To count these full and partial cubes, we need to use the formula for the area of a circle.

The radius of the circle base (bottom) is 3 units and the formula for the area of a circle is $A = \pi r^2$. So the number of cubes is approximately $(3.14)(3)^2 = (3.14)(9)$, which to the nearest tenth, is equal to $28.3\ units^2$.

If we imagine the cylinder like a building, we could stack cubes on top of each other until the cylinder is completely filled. It would be filled so that all the cubes are touching each other such that no space existed between cubes.

It would look like this.



To count all the cubes above, we will use the consistency of the solid to our advantage.

We already know that there are 28.3 cubes on the bottom level and all levels contain the exact same number of cubes.

Therefore, we need only take the bottom total of 28.3 and multiply it by 4 because there are four levels to the cylinder.

$(28.3\ u^3)(4) = 113.2$ total cubic units in our original cylinder.

"B" h

To understand the units of our answer, we could think in terms of algebra and exponents. We know that $(x)^2$ *times* x *is* $x^2 \cdot x^1$, which equals x^3, similarly, $(units)^2$ *times* $units^1 = units^3$ for the same reason.

x^{2+1} $units^{2+1}$

So if we had to find the volume of our original cylinder, all we needed to do was multiply π times $(3\ units)^2\ (4\ units)$ **to get $V \approx 113.2\ units^3$.**

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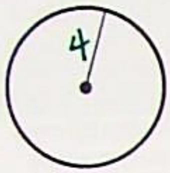
Unit 9 Geometry | Volume of Cylinders

NOTES "B" – area of the base face ○○○ circle

To find the volume of a cylinder, you will need to recall how to calculate the area of a circle!

Find the area of each circle. Use the formula $A = \pi r^2$. **Write your answers in terms of π.**

1) Radius = 4 cm



$A = \pi r^2$

$\pi(4)^2$

$\boxed{16\pi\, cm^2}$

$B = 16\pi$

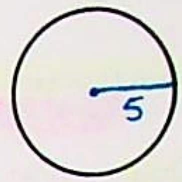
2) Diameter = 6 ft

$A = \pi(3)^2$

$\boxed{9\pi\ ft^2}$

$B = 9\pi$

3) Circumference = 10π u



$C = \pi D \quad C = 10\pi$

$D = 10$

$r = 5$

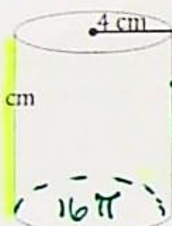
$A = \pi(5)^2$

$\boxed{25\pi\, u^2}$

$B = 25\pi$

Use your answers to questions 1 – 3 to calculate the volume of the cylinders below. **Write your answers in terms of π and then round to the nearest tenth.** $V = Bh$

4) Radius = 4 cm



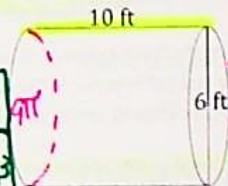
$V = Bh$

$(16\pi)(5)$

$\boxed{V = 80\pi\, cm^3}$

$\boxed{V \approx 251.3\, cm^3}$

5) Diameter = 6 ft

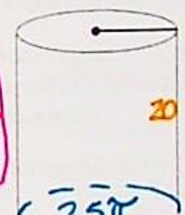


$V = (9\pi)(10)$

$\boxed{V = 90\pi\, ft^3}$

$\boxed{V \approx 282.7\, ft^3}$

6) Circumference = 10π u



$V = Bh$

$= (25\pi)(20)$

$= 500\pi\, u^3$

$\approx 1570.8\, u^3$

The **FORMULA** for the volume of a cylinder is: $V = \pi r^2 h$. $V = Bh$

Where "r" is the radius of the circular face at the base of the cylinder, and "h" is the height of the cylinder.

To find the volume of the cylinder to the right, substitute the measurements into the formula above.

Notice that in this figure, the diameter is given, and we need the radius. $r = 4\,cm$

Diameter = 8 radius, so r = 4

Height of the cylinder = 9

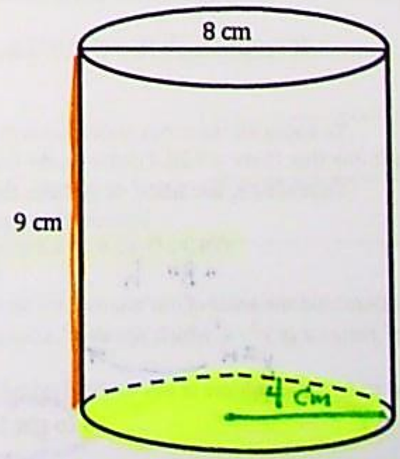
Formula: $V = \pi r^2 h$

$V = \pi(\underbrace{4}\,)^2(9) = \pi(16)(9)$

144π

$V = 144\pi\, cm^3$

$V \approx 452.4\, cm^3$



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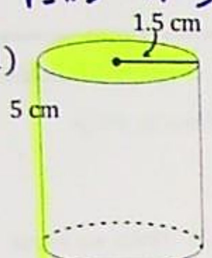
~~**Assignment**~~ (More Notes/Guided Practice!)

Find the volume of the cylinders below. Be sure to include cubic measurements in your answers and leave your answers in terms of π and then round to the nearest tenth.

Use the **FORMULA**: $V_{cylinder} = \pi r^2 h.$

$r=1.5 \quad h=5$

1)



$V = \pi r^2 h$

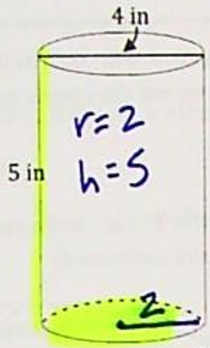
$\pi(1.5)^2(5)$

$\pi(2.25)(5)$

$V = 11.25\pi\ cm^3$

$V \approx 35.3\ cm^3$

2)



$r=2$

$h=5$

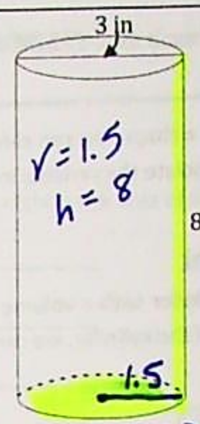
$V = \pi(2)^2(5)$

$\pi(4)(5)$

$V = 20\pi\ in^3$

$V \approx 62.8\ in^3$

3)



$r=1.5$

$h=8$

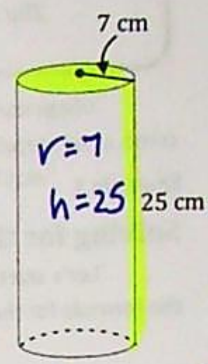
$V = \pi(1.5)^2(8)$

$\pi(2.25)(8)$

$V = 18\pi\ in^3$

$V \approx 56.5\ in^3$

4)



$r=7$

$h=25$

$V = \pi(7)^2(25)$

$\pi(49)(25)$

$V = 1225\pi\ cm^3$

$V \approx 3848.5\ cm^3$

5) Radius = 10 m
Height = 4 m

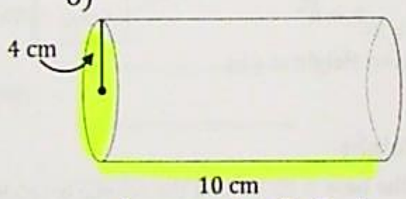
$V = \pi(10)^2(4)$

$= \pi(100)(4)$

$V = 400\pi\ m^3$

$V \approx 1256.6\ m^3$

6)



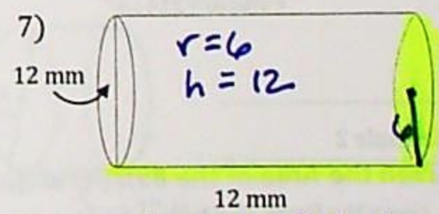
$V = \pi(4)^2(10)$

$\pi(16)(10)$

$V = 160\pi\ cm^3$

$V \approx 502.7\ cm^3$

7)



$r=6$

$h=12$

$V = \pi(6)^2(12)$

$\pi(36)(12)$

$V = 432\pi\ mm^3$

$V \approx 1357.2\ mm^3$

8) Circumference = 8π in

$\boxed{h=12}$

$C = 8\pi \quad C = \pi D$

$D = 8 \quad \boxed{r=4}$

$\pi(4)^2(12)$

$\pi(16)(12)$

$V = 192\pi\ in^3$

$603.2\ in^3$

9) Circumference = 5π ft

$\boxed{h=14}$

$D=5$

$\boxed{r=2.5}$

$\pi(2.5)^2(14)$

$\pi(6.25)(14)$

$V = 87.5\pi\ ft^3$

$V \approx 274.9\ ft^3$

Notes - VOLUME OF 3-D FIGURES - Right Prisms and Cylinders; finding any dimension

To find the volume of a right prism or cylinder, multiply the Base Area by the height.

BASIC VOLUME FORMULA FOR ANY RIGHT PRISM: $V = Bh$

*The Volume formula for a **CYLINDER** is:* $(\pi r^2)h$

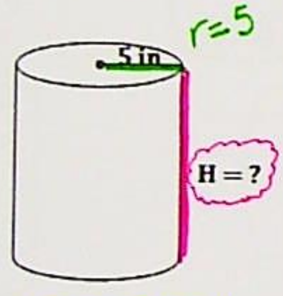
Area of the base

Given the volume of a shape, we can solve for a missing dimension such as the height or radius. It should come as no surprise that to isolate the variable in the equation, we will use inverse operations.

Example 1

Solving for the Height

Let's start with a cylinder with a volume of approximately 314 in^3 and a radius of 5 in. Since we know the formula for the volume of the cylinder, we can plug in and work backwards.



Volume ≈ 314 in^3

$V = \pi r^2 h$

Substitute what we know

$314 = 3.14 \cdot (5)^2 h$

$314 = 3.14 \cdot 25 \cdot h$

Simplify

$314 = 78.5h$

$\frac{314}{78.5} = \frac{78.5h}{78.5}$

Solve

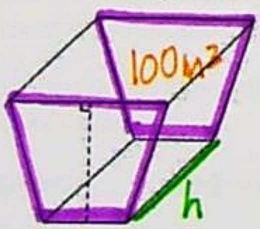
$4 = h$

Answer: Height = 4 in

Example 2

Given the Area of the Base, find the Height

Given the figure. find the height if the area of the base is 100m² and the volume is 1200m³.



Identify the base by name: Trapezoid

Show work

$V = Bh$

$\frac{1200}{100} = \frac{\cancel{100}h}{\cancel{100}}$

$\boxed{h = 12\text{ m}}$

$B = 100$

$V = 1200$