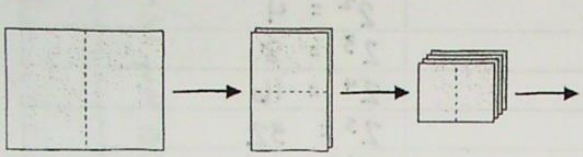
~~ Unit 5, Page 4 ~~

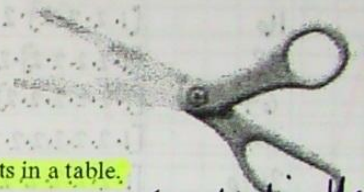
Objectives: I can model an exponential relationship with a function table and graph.

Growing, Growing, Growing: Investigation 1

Chen, the secretary of the Student Government Association, is making ballots for tonight's meeting. He starts by cutting a sheet of paper in half. He then stacks the two pieces and cuts them in half. He stacks the resulting four pieces and cuts them in half. He repeats this process, creating smaller and smaller pieces of paper.

Begin
1 ballot
(zero cuts)





After each cut, Chen counts the ballots and records the results in a table.

Number of Cuts (x)	Number of Ballots (y)
0	1
1	2
2	4
3	8
4	16

(extending the table)

cuts	ballots
5	32
6	64
7	128
8	256
9	512
10	1024

Chen wants to predict the number of ballots after any number of cuts.

1. Predict how many ballots will result from 3 cuts. 8
2. Predict how many ballots will result from 4 cuts. 16
3. Predict how many ballots will result from 10 cuts. 1024

4. Complete the 2nd column in the table to show the number of ballots after each of the cuts.

Number of Cuts (n)	Number of Ballots (b)	Calculations for Number of Ballots (b)	Shortcut Form for Number of Ballots using Exponents (b)
0	1	1	$2^0 = 1$
1	2	[1] · 2	$2^1 = 2$
2	4	[1 · 2] · 2	$2^2 = 4$
3	8	[1 · 2 · 2] · 2	$2^3 = 8$
4	16	[1·2·2·2]·2	$2^4 = 16$
5	32	[1·2·2·2·2]·2	$2^5 = 32$
6	64	[1·2·2·2·2·2]·2	$2^6 = 64$
7	128	[1·2·2·2·2·2·2]·2	$2^7 = 128$
8	256	[1·2·2·2·2·2·2·2]·2	$2^8 = 256$
9	512	[1·2·2·2·2·2·2·2·2]·2	$2^9 = 512$
10	1024	[1·2·2·2·2·2·2·2·2·2]·2	$2^{10} = 1024$

5. How did you find your entries in the table? each additional cut meant another factor of 2, which doubled the previous amount

6. Fully complete the table above.

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What is the relationship between the number of ballots and the previous number of ballots?

the number of ballots is double the previous amount

7. What is the relationship between the number of cuts and the number of ballots? (In other words, how can you use the number of cuts to figure out the number of ballots?)

The number of cuts, n, is the exponent on a base of 2, so the equation: $b = 2^n$ models the situation.

8. A rule (equation) to explain the relationship between of the number of cuts (n) and the number of ballots (b) is $= 1 * 2^n$. Use this rule (equation) to determine how many ballots Chen would have if he made 20 cuts? $n = 20$

Show your work. $b = 1 * 2^n = 1 * 2^{20} = 1 * 1{,}048{,}576 = 1{,}048{,}576$

9. Use the rule (equation) to determine how many ballots Chen would have if he made 30 cuts? $n = 30$

Show your work. $b = 1 * 2^n = 1 * 2^{30} = 1 * 1{,}073{,}741{,}824 = 1{,}073{,}741{,}824$

$b = 500$

10. How many cuts would it take to make enough ballots for all 500 students in Chen's school? $n = 9$

Explain how you determined this answer. Substitute 500 for b, then guess & check to find n (the exponent)... or use the table above.

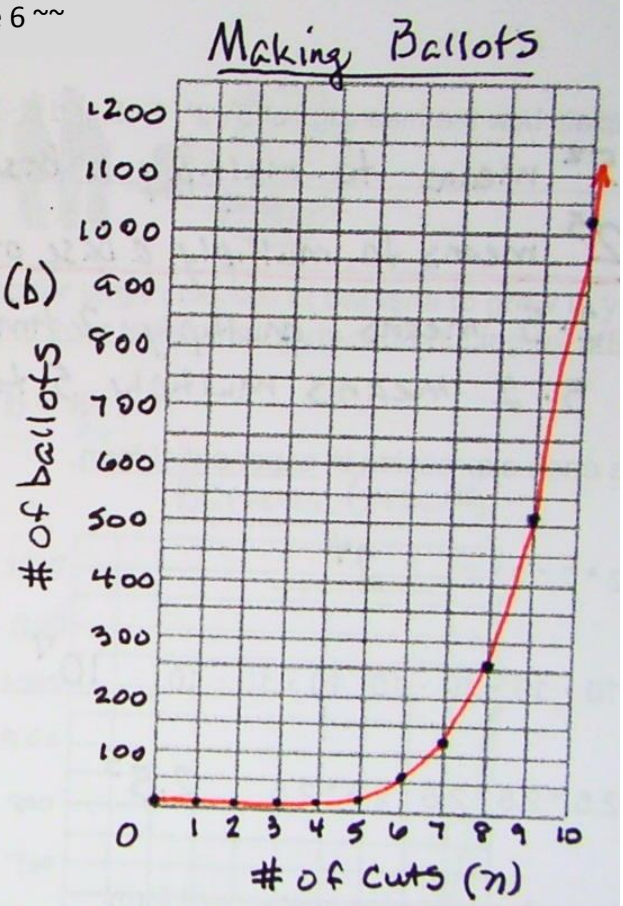
$2^n = 500$, $2^9 = 512$, so $n = 9$

12. Graph the relationship.

Use an interval of 1 on the x-axis and 50 on the y-axis.

# of cuts n	# of ballots b
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1024

(b)



When you found the number of ballots after 10, 20 and 30 cuts, you may have multiplied a long string of 2s. Instead of writing long product strings of the same factor, you can use the exponential form. For example you can write $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ as 2^5, which is read as "2 to the fifth power."

In the expression 2^5, you get $2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$. We say that 32 is the standard form for 2^5.

13. Write each expression in exponential form.

a) $2 \cdot 2 \cdot 2$ 2^3

b) $5 \cdot 5 \cdot 5 \cdot 5$ 5^4

c) $1.5 \cdot 1.5 \cdot 1.5 \cdot 1.5 \cdot 1.5 \cdot 1.5 \cdot 1.5$ 1.5^7

14. Write each expression in standard form.

a) 2^7 $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 128$

b) 3^3 $3 \cdot 3 \cdot 3 = 27$

c) 4.2^3 $(4.2)(4.2)(4.2) = 74.088$

15. Most calculators have a [^] or [y^x] key for evaluating exponents. Use your calculator to find the standard form for each expression.

a) 2^{15} 32,768

b) 3^{10} 59,049

c) 1.5^{20} 3325.25673