

Unit 8 Geometry | Solids

Rectangular Prisms

Measurement of volume is expressed in cubic units such as in^3 , ft^3 , m^3 , cm^3 , or units^3. The volume of a solid is the number of cubic units that can be contained in the solid.

First, let's look at a rectangular solid.



PRACTICE EXAMPLES

Find the volume of the block figures below. Use the formula $V = _$



8 in

EXAMPLE 2:

In some problems you must *use the volume* to find another missing measure.

- 1) Find the *length* of the prism if the volume of the prism is $84 m^3$
- 2) Find the *width* of the square prism if the volume of the box is $100 in^3$.

w in



3) The area of the base of the shark cage is 42 ft^2 , and the volume of the cage is 378 ft^3 . Find the <u>height</u>



w in

packaging there is these days."

Assignment

1. Determine the volume of the rectangular prism shown below.



2. The volume of the prism shown below is 61.6 in³. What is the height of the prism?



3. The volume of the prism shown below is 972 cm³. What is its length?



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 The volume of the prism shown below is 32.7375 ft³. What is its width?



5. Determine the volume of the 3-dimensional figure below. Explain how you got your answer.



6. Find the value of the ratio that compares the volume of the larger prism to the smaller prism.



(Assignment cont'd) Multiple Choice Questions

 Jerry poured water to a height of 1¹/₂ feet into a new aquarium with dimensions shown below.



One cubic foot of water weighs approximately 62 pounds. What is the weight of the water that Jerry put in the aquarium?

- A. 403 pounds B. 465 pounds
- C. 558 pounds D. 744 pounds

8) The formula for the volume (V) of a cube is

$$V = e^3$$

where e is the length of an edge.

An edge of a silver cube is twice as long as an edge of a gold cube. How many times greater is the volume of a silver cube than that of a gold cube?

- A. 2 times greater B. 9 times greater
- C. 8 times greater D. 6 times greater



This first stack of blocks gets knocked down. Four of these blocks are then used to begin a second stack with the base shown above. How high will the second stack be if all the blocks are used?

A.	4 blocks	В.	6 blocks

- C. 8 blocks D. 10 blocks
- 10) A bank teller fills 3 drawers with stacks of \$20 bills. The dimensions of each stack are approximately 3 cm × 6 cm × 15 cm. The dimensions of one drawer are



Note: The figures are not drawn to scale.

How many stacks will fill 3 drawers?

A.	60 stacks	В.	80 stacks

- C. 240 stacks D. 270 stack9)
- All the boxes are the same size. Linda has three different sizes of balls as shown in the picture below.



If she fills each box with the kind of balls shown, which box will have the fewest balls in it?

- A. The box with the tennis balls.
- B. The box with the golf balls.
- C. The box with the rubber balls.
- D. Each box will have the same number of balls.

12) Use the cubes below to answer the following question.



Which of the following correctly describes the volume of cube B compared to the volume of cube A?

- A. The volume of cube B is 2 times the volume of cube A.
- B. The volume of cube B is 4 times the volume of cube A.
- C. The volume of cube B is 6 times the volume of cube A. $\label{eq:cube}$
- D. The volume of cube B is 8 times the volume of cube A.
- The base of the rectangular prism shown below is a 3-foot square.





What is the volume of the prism?

- A. 108 cubic feet B. 144 cubic feet
- C. 324 cubic feet D. 432 cubic feet
- These two cubes have sides 2 cm and 4 cm, respectively.



What is the ratio of their volumes?

A.	1 to 2	B.	1 to 4
C.	1 to 8	D.	1 to 16

Calculating the Volume of Right Prisms

To find the volume of any right prim, calculate the area of the BASE and multiply by the height.

V = Bh, where B is the area of the base.

Examples

1.	Identify the polyhedron by name	2:	
		Identify the base by name:	
		Calculate the area of the base:	
		Show work	
	12 m 4 m	Calculate the volume:	
2.	Identify the polyhedron by name	2:	
		Identify the base by name:	
		Calculate the area of the base:	
/		Show work	
13 mm	13 mm	Calculate the volume:	
3.	Identify the polyhedron by name	2:	
	5 cm	Identify the base by name: Calculate the area of the base:	
/	E	Show work	
	9 cm 6 cm	Calculate the volume:	

Assignment

1. Identify the polyhedron by name: ______



Identify the base by name:				
Calculate the area of the base:				
Show work				
Calculate the volume:				

2. Identify the polyhedron by name: ______



Identify the base by name: _____ Calculate the area of the base: _____ Show work _____ Calculate the volume: _____

3. Identify the polyhedron by name: ______



Identify the base by name: ______ Calculate the area of the base:

Show work



Calculate the volume:

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General Cylinder

Specific Cylinder

Did you potice?

Just like with rectangular prisms, every cross section of a cylinder is the same as

the bases. With

cylinders the cross sections are all

congruent:

Volume of Cylinders Explained

The process for understanding and calculating the volume of cylinders is identical to that of **prisms**, even though cylinders are curved. **V = Bh** Here is a general **cylinder**. The base is a _____.

Let's start with a specific cylinder with radius 3 units and height 4 units.

We will **fill the bottom** of the cylinder with unit cubes. This means the bottom of the prism will act as our **surface** and will be **covered** with as many unit cubes as possible without stacking them on top of each other yet.

This is what it would look like.

The diagram above is strange looking because we are trying to stack cubes within a curved space. Some cubes have to be shaved so as to allow them to fit inside. Also, the cubes do not yet represent the total volume. It only represents a partial volume, but we need to count these cubes to arrive at the total volume.

To count these full and partial cubes, we need to use the formula for the area of a circle.



To count all the cubes above, we will use the consistency of the solid to our advantage. We already know that there are 28.3 cubes on the bottom level and all levels contain the exact same number of cubes. **Therefore, we need only take the bottom total of 28.3 and multiply it by 4** because there are four levels to the cylinder.

 $(28.3 \text{ u}^3)(4) = 113.2 \text{ total cubic units in our original cylinder.}$

To understand the units of our answer, we could think in terms of algebra and exponents. We know that $(x)^2$ times x is $x^2 \cdot x$, which equals x^3 , similarly, $(units)^2$ times units = units³ for the same reason.

So if we had to find the volume of our original cylinder, all we needed to do was multiply π times (3 units)² (4 units) **to get V** \approx **113.2 units**³.

Unit 9 Geometry | Volume of Cylinders

<u>NOTES</u>

To find the volume of a cylinder, you will need to recall how to calculate the area of a circle!

Find the <u>area</u> of each circle. Use the formula $A = \pi r^2$. Write your answers in terms of π .



Use your answers to questions 1 - 3 to calculate the <u>volume</u> of the cylinders below. Write your answers in terms of π and then round to the nearest tenth.



Where "r" is the ______ of the circular face at the base of the cylinder, and "h" is the ______ of the cylinder.







8) Circumference = 8π in, height = 10 in

9) Circumference = 5π ft, height = 20 ft

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Notes -	VOLUME	OF 3-D F	IGUKES -	· Kight Prisms	and Cylinde	ers; finding an	y aimension

To find the volume of a right prism or cylinder, multiply the _____ by the _____.

BASIC VOLUME FORMULA FOR ANY RIGHT PRISM: _____

The Volume formula for a **CYLINDER** is:

Given the volume of a shape, we can solve for a missing dimension such as the height or radius. It should come as no surprise that to isolate the variable in the equation, we will use inverse operations.

Example 1

Solving for the Height

Let's start with a cylinder with a volume of approximately 314 in³ and a radius of 5 in. Since we know the formula for the volume of the cylinder, we can plug in and work backwards.



Example 2

Given the Area of the Base, find the Height

Given the figure. find the height if the area of the base is 100m² and the volume is 1200m³.



Identify the base by name: _____

Show work

Examples for you to try:

3) The volume of a cylinder is 405π with a diameter of 18. Find the height of the cylinder.



4) The volume of the triangular prism is 312cm³. Find the height is the area of the base is 52cm².



Some of the problems in the assignment will be review, solving for the volume of the figure.

Assignment:

FIND THE VOLUME. Show all work.

1)	4 cm 11 cm	2)	18 in.	
	In terms of π :		In terms of π :	
	Volume =		Volume =	_
	Volume to the nearest tenth. \approx		Volume to the nearest tenth	.≈

3) The volume of a cylinder is 450 π with a radius of 10. Find the height of the cylinder. Show all work.



- **4)** Find the volume. Show all work.



Find the volume of each solid. Show all work.



Find the **indicated dimension** of each <u>**PRISM**</u>. Show all work.



9) The volume of a cylinder is approximately 5626.9 ft³ with a diameter of 32 ft. Find the height of the cylinder. Show all work.



10) Find the volume. Show all work.



11) Find the height. Show all work.



Area of Base: 45 cm²

Volume: 360 cm³

12) Find the height. Show all work.



Area of Base: 58 m^2

Volume: 174 m³

Volume of Pyramids

We will be finding the volume of rectangular and triangular right pyramids. A pyramid is considered to be right if the apex(top) is directly above the center of the base.

Rectangular Pyramid

h

a

Н

Triangular Pyramid

Square Pyramid

A pyramid is...

- A solid object where:
- The base is a polygon (a straightsided flat shape)
- The sides are triangles which meet at the top (the apex).

It is a polyhedron.

Remember that a **prism** has two congruent bases with rectangular sides.

Describe the following polyhedron by identifying the base and if it is a pyramid or prism.



Find the volume. Round to the nearest tenth as needed.



Find the missing dimension.



 $Volume = 15360 \, ft^3$

<u>Assignment</u>

Part A) Find the volume or missing dimension of each pyramid. Show all work.



Volume = ?







Volume = 5440 ft^3

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Part C) Find the **volume** of the prism, pyramid or cylinder. Identify the name first. State the formula to be used. Show and label all work. If π is used, state your answer first in terms of π , then use 3.14 for π and round to the nearest tenth. (Some dimensions are given that you won't need to use.)



Volume of Cones

NOTES: To find the volume of a cone, substitute the measurements given for the cone into the correct formula and solve. Remember, volumes are expressed using cubic units, such as in^3 , ft^3 , m^3 , cm^3 , or units^3.

Volume of a CONE

FORMULA:
$$V_{cone} = \frac{1}{3}\pi r^2 h$$



In words: The volume of a cone equals one-third the volume of a cylinder with the same radius and height!



Find the volume of the following cones. Leave answers in terms of π , then approximate to the nearest tenth using 3.14 for π .







Multiple Choice (Show work.)

7) A cone has a height of 4 inches and a circumference at the base of 12π inches.

What is the approximate volume of the cone?

- A. 150 inches³ B. 200 inches³
- C. 450 inches³
 D. 600 inches³

8) Cecil has a paper cup in the shape of a cone, as shown below.

Cecil's Paper Cup



What is the volume of Cecil's paper cup?

- A. $2\frac{1}{2}\pi$ cubic in. B. $3\frac{3}{4}\pi$ cubic in.
- C. 15π cubic in. D. 60π cubic in.

Review

Find the volume of each figure. If necessary, round to the nearest tenth.



Volume of Spheres

Definition

Sphere – the set of all points in space that are the same distances from a center point.

FORMULA:
$$V_{sphere} = rac{4}{3} \pi r^3$$

<u>Part A</u>) For Examples 1 and 2, find the volume of each sphere.

Part B) HEMISPHERES

<u>Definition</u>

HEMISPHERE – a circular cross section that separates a sphere into two congruent halves.

Example 1:

Find the volume of the hemisphere with a diameter of 15 km. Round to the nearest tenth.

Example 2:

The inside of a cereal bowl is in the shape of a hemisphere. Find the maximum amount of milk that can fit in the bowl. *Round to the nearest hundredth*.

Part C) DETERMINING MISSING LENGTHS

Example 1:

The volume of a golf ball is about 13.2π cm³. What is the **radius** of the golf ball to the nearest tenth?

Example 2:

The volume of a baseball is about **13.39** cubic inches. What is the **diameter** of the baseball to the nearest tenth?

Assignment

Find the exact volume (leave the answer in terms of π). Then use 3.14 for π and round to the nearest tenth. Show all work.

10) The volume of a sphere is 288π ft³. What is the diameter of the sphere?

11) The volume of a sphere is about 310.2 cm³. What is the approximate radius?

15 cm

Volume of Composite or Combined Figures

Notes

Find the volume of each figure. Round the answer to two decimal places. (use $\pi = 3.14$)

Label your work. Identify the figure name, write the formula and show all work.

1) $\frac{12 \text{ ft}}{16 \text{ ft}}$	2)
Name:	Name:
Formula:	Formula:
Name:	Name:
Formula:	Formula:
Combined Volume:	Combined Volume:

Assignment

Find the volume of each figure. Round the answer to two decimal places. (use $\pi = 3.14$)

Label your work. Identify the figure name, write the formula and show all work.

1.			2.
	ED B cm	2011	uig uig 14 in
	Name:		Name:
	Formula:		Formula:
	Name:		Name:
	Formula:		Formula:
			Combined Volume:
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5 mm um 5 mm 4 mm 4	5.
Name:	Name:
Formula:	Formula:
 Name: Formula:	
Combined Volume:	Combined Volume:
G. Formula:	Name: Formula:
ő in	

Carefully read and solve the problems below.

Solid Geometry Word Problems

<u>NOTES</u>

1. Robert is using a cylindrical barrel filled with water to flatten the sod in his yard. The circular ends of the barrel have a radius of 1 foot. The barrel is 3 feet tall. How much water will the barrel hold?

Find the volume formula for a CYLINDER on your reference sheet and record below. Formula:

2. If a basketball measures 24 centimeters in diameter, what volume of air will it hold?

Find the volume formula for a SPHERE on your reference sheet and record below. Formula:

3. What is the volume of a sugar cone that is 2 inches in diameter and 5 inches tall?

Find the volume formula for a CONE on your reference sheet and record below. Formula:

Practice

Find the volume of each solid. Show all work.

1) Approximately how much air would be needed to fill a dozen soccer balls with a radius of 14cm? *Round to the nearest hundredth.*

2) Find the volume of the following figure if the diameter is 4.5 in and the height of the cylinder is 2.5 in. *Round to the nearest tenth.*

3) The diameter of the earth is approximately 7,926 miles. The diameter of the moon is approximately 2,159 miles. Approximately how many moons would fit inside the earth?

4) Find the radius of a sphere with a volume of 1,767.1 m³. *Round to the nearest tenth.*

5) Find the radius of a hemisphere with a volume of 2,712.3 in³. *Round to the nearest tenth.*

Review

1. Find the difference between the volumes of the two objects below.

2. Find the volume of the compound figure below.

Directions: Find the volume of the following figures and situations.

Find the volume to the nearest tenth.

Volume ≈ _____

6) Find the volume of a cylindrical cake that is 5 in. tall with a radius of 7.5 in.

7) A standard men's basketball has a circumference of about 29.5 inches. What is the volume of the basketball to the nearest hundredth? (*hint: find the diameter first.*)

8) A cylindrical container is used to hold dog food. Its volume is approximately 50.27 ft³ and has a radius of 2 ft. What is the height of the container to the nearest foot?

5)

9) A globe in a brass stand has an approximate volume of 33,510.32 in³, what is its radius length?

10) Find the volume.

11) Find the volume, to the nearest tenth, of a 4 ft by 2 ft by 3 ft rectangular prism with a cylindrical hole, radius 6 in., through the center.

12) Marge has a cylindrical tin of popcorn that is 18 in. tall and has a radius of 4 in. She wants to use the tin for something else and needs to empty the popcorn into a box. The box is 8 in. long, 8 in. wide and 14 in. tall. Will the popcorn fit in the box? Explain.

13) Spaceship Earth at Epcot Center in Florida is a 180-foot geosphere. Find the volume by assuming it is a sphere with a diameter of 180 feet.

14) The volume of the following soup can is 22π in³, and has a height of 5.5 in. What is the radius of the soup can?

15) Based on the following drawing, if the top funnel was filled with water and then emptied into the bottom cone, what fraction of the bottom cone would be filled with water? Explain.

16) A cylinder is 9 inches high. The circumference of the base is 12π inches. Find the volume.

17) The height of a cylinder is 10 and the area of a base is 36π square units. What is the volume in cubic units?

18) A can of soup contains about 553 cubic centimeters of soup. The height of the can is 11 cm. What is the approximate diameter of the can to the nearest centimeter?

19) A scented candle is in the shape of a cylinder, with a radius of 4cm and a height of 12cm.

Find the volume (leave in terms of π).

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20) A cylindrical cake takes up 32π cubic inches. The diameter of the cake is 8 inches, what is the height of the cake?

21) Nate uses a cube shaped bead with side lengths measuring 6mm. Each bead has a circular hole in the middle. The diameter of the circular hole is 3mm. Find the volume of the bead.

[not drawn to scale]

In terms of π , Volume = _____

Now find the volume to the nearest tenth.

Volume ≈ _____

22) If the volume of a cube is 729 cubic feet, then what is the length of one edge of the cube?

23) Multiple Choice

Find the volume of concrete used to construct the ramp.

A) 30 <i>ft</i> ³	C)	66	ft³
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B) $36 ft^3$ **D)** $96 ft^3$

24) The volume of a cylinder is about 1632 in³. The height of the cylinder is 24in. What is the area of the base?

25) Tanya uses a cube shaped bead with side lengths measuring 12mm. Each bead has a circular hole in the middle. The diameter of the circular hole is 2mm. Find the volume of the bead. [*Hint: use the diagram*.]

26) A chocolate bar is in the shape of a trapezoidal prism as shown below. Find the volume of the chocolate bar.

27) Multiple Choice

Find the maximum amount of water that can fill the trough shown.

28) Multiple Choice

What is the volume of a cylinder with a radius of 8 inches and a height of 1 foot? Round answer to the nearest tenth.

29) Tennis balls with a diameter of 3 inches are sold in cans of three. The can is in the shape of a cylinder. What is the volume of the space NOT occupied by the tennis balls? Assume the tennis balls touch the can on the sides, top and bottom. *Round your answer to the nearest tenth.*

 Marny wants to approximate the amount of wax needed to make a crayon. The dimensions of the crayon are shown below.

What is the volume left in the cylinder after the shaded cone region is removed?

A.	$1,024\pi \mathrm{in.^3}$	В.	$1,536\pi \text{ in.}^3$
C.	2.048π in. ³	D.	$4.096\pi \mathrm{in.}^3$

About how many cubic centimeters of wax are needed to make this crayon?

- A. 18 cm³ B. 22 cm³
- C. 28 cm³ D. 88 cm³
- **35.** The volume of a cone is 405π in³ with a diameter of 18in. Find the height of the cone.

36. An ice cream shop designs a new ice cream cone. He wants the volume to be about 240cm³. The cone is 14cm tall. What is its radius to the nearest whole number?

