Name:_____ Date:_____

Geometry Worksheet: 3.4 Beyond CPCTC

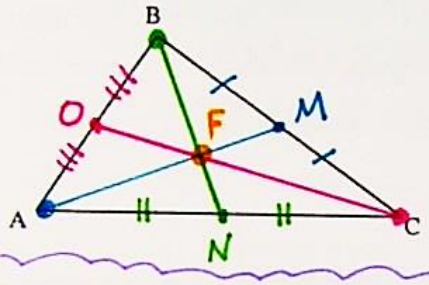
Defn: **median** – a line segment drawn from any vertex of a triangle to the midpoint of the opposite side.

For proof reasons

COND: If a segment is a median, then it bisects the side to which it is drawn.

CONV: If a segment drawn from a vertex of a triangle bisects the side to which it is drawn, then it is a median.

1. Draw 3 medians – one from A to $\overline{BC}$, one from B to $\overline{AC}$ and a third from C to $\overline{AB}$.
   Label the 3 midpoints, M, N and O, respectively.
   Use tick marks to show congruent segments.



Centroid
point F

Note: The point where all 3 medians intersect is called the *centroid*.
Label the centroid as point F.

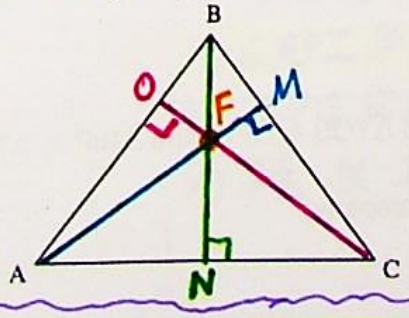
Defn: **altitude** – a line segment drawn from any vertex of a triangle such that it is perpendicular to the opposite side or the extension thereof.

For proof reasons

COND: If a segment is an altitude, then it forms right angles with the side to which it is drawn.

CONV: If a segment drawn from a vertex of a triangle forms right angles with the side to which it is drawn, then it is an altitude.

2. Draw 3 altitudes – one from A to $\overline{BC}$, one from B to $\overline{AC}$ and a third from C to $\overline{AB}$.
   Label the 3 points of intersection as M, N and O, respectively.
   Use proper tick marks to show right angles.

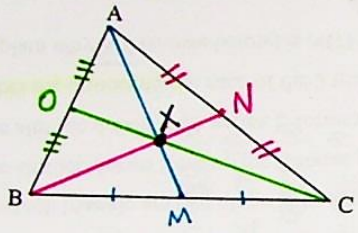


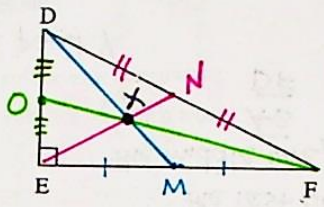
Orthocenter
point F

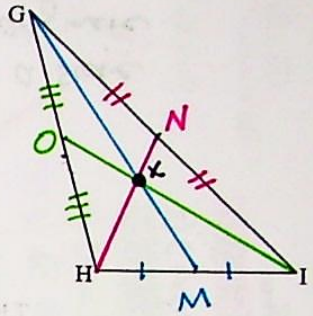
Note: The point where all 3 altitudes intersect is called the *orthocenter*.
Label the orthocenter as point F.

3.4 Medians

Draw the 3 medians for each triangle below. Use a ruler for accuracy.



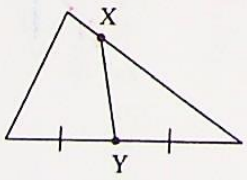




(8)

Questions:

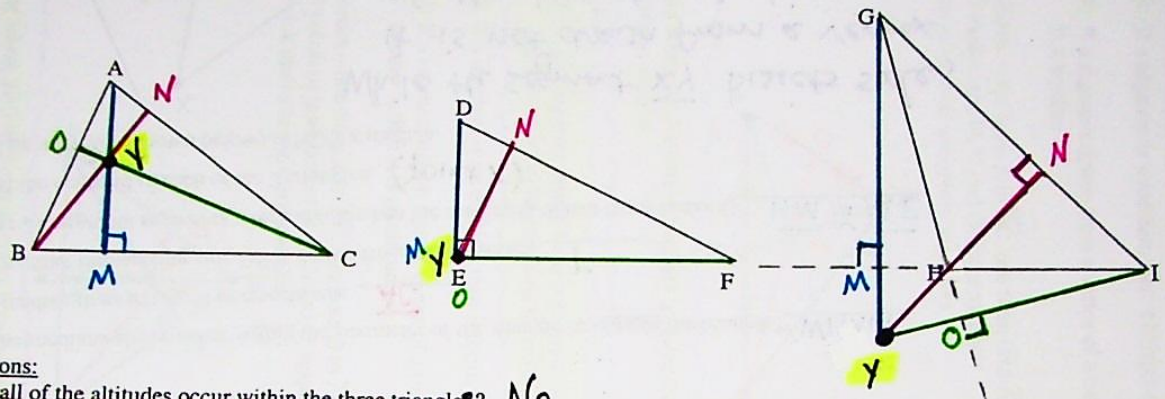
1. Do the medians above occur within the perimeter of the triangle or outside the triangle? within
2. The median drawn from B intersects side $\overline{AC}$.
3. The median intersecting side $\overline{DE}$ is drawn from which vertex F.
4. Name 2 congruent segments which result when the median is drawn from vertex G: $\overline{HM} \cong \overline{MI}$
5. Label the centroid in each of the 3 triangles. (point X)
6. Explain why $\overline{XY}$ (shown below) is **NOT** a median.



While the segment $\overline{XY}$ bisects side, it is not drawn from a vertex of the triangle. Medians originate from a vertex of a triangle.

3.4 Altitudes

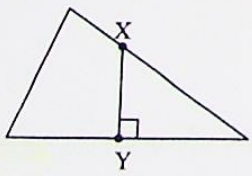
Draw the 3 altitudes for each triangle below. Use a ruler for accuracy.



(9)

Questions:

1. Do all of the altitudes occur within the three triangles? No
2. In which triangle do two of the altitudes fall outside the triangle? the last one, ΔGHI
3. In which triangle are two of the altitudes on the triangle? the middle one, ΔDEF
4. The altitude drawn from vertex C forms rights angles with which side? $\overline{AB}$
5. The altitude drawn from vertex E forms rights angles with which side? $\overline{DF}$
6. Label the orthocenter in each of the 3 triangles. point Y
7. Explain why $\overline{XY}$ (shown below) is **NOT** an altitude.



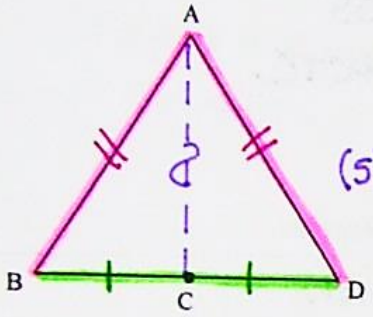
While $\overline{XY}$ forms a right $\angle$ on a side of the triangle, it does not originate from a vertex of the triangle. Altitudes originate from a vertex of a triangle and form right angles on opposite side.

Defn: **auxiliary line** – a line added to a diagram to help complete a proof. (*dotted*)

Postulate: any two points determine a line.

3. Given: $\overline{AB} \cong \overline{AD}$
C midpoint $\overline{BD}$

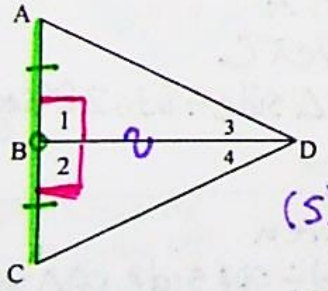
Prove: $\angle B \cong \angle D$



Statements	Reasons
1) $\overline{AB} \cong \overline{AD}$	1) Given
2) C mdpt $\overline{BD}$	2) Given
3) $\overline{BC} \cong \overline{CD}$	3) A mdpt ÷ seg into 2 $\cong$ segs
4) Draw $\overline{AC}$	4) 2 pts determine a seg
(5) 5) $\overline{AC} \cong \overline{AC}$	5) Reflexive Property
6) $\triangle ABC \cong \triangle ADC$	6) SSS (1, 3, 5)
7) $\angle B \cong \angle D$	7) CPCTC

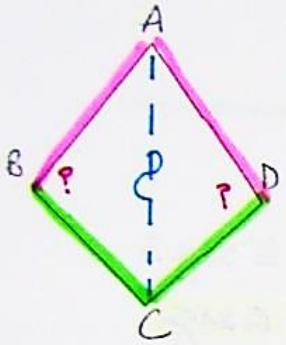
4. Given: $\overline{DB}$ is an altitude to $\overline{AC}$
B midpoint $\overline{AC}$

Prove: $\overrightarrow{DB}$ bisects $\angle ADC$



Statements	Reasons
1) $\overline{DB}$ alt to $\overline{AC}$	1) Given
2) $\angle 1$ & $\angle 2$ Rt $\angle$s	2) alt forms Rt $\angle$s on opp side of $\triangle$
3) $\angle 1 \cong \angle 2$	3) All right $\angle$s are $\cong$
4) B mdpt $\overline{AC}$	4) Given
5) $\overline{AB} \cong \overline{BC}$	5) A mdpt ÷ seg into 2 $\cong$ segs
(5) 6) $\overline{BD} \cong \overline{BD}$	6) Reflexive Property
7) $\triangle ABD \cong \triangle CBD$	7) SAS (5, 3, 6)
8) $\angle 3 \cong \angle 4$	8) CPCTC
9) $\overrightarrow{DB}$ bis $\angle ADC$	9) If $\angle$ is ÷ into 2 $\cong$ $\angle$s by a ray, then bisected.

1. Now write the proof for it:

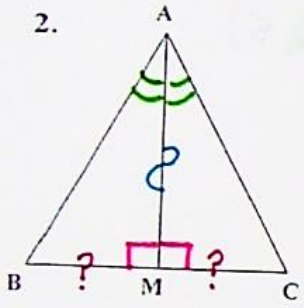


Given: $\overline{AB} \cong \overline{AD}$, $\overline{BC} \cong \overline{DC}$

Prove: $\angle B \cong \angle D$

Statements	Reasons
1) $\overline{AB} \cong \overline{AD}$	1) Given
2) $\overline{BC} \cong \overline{DC}$	2) Given
3) Draw $\overline{AC}$	3) 2 pts det a seg
4) $\overline{AC} \cong \overline{AC}$	4) Reflexive Property
5) $\triangle ABC \cong \triangle ADC$	5) SSS
6) $\angle B \cong \angle D$	6) CPCTC

2.

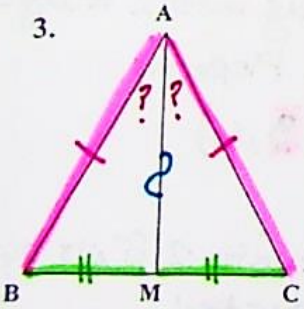


Given: $\overline{AM}$ is an altitude, $\overrightarrow{AM}$ bis. $\angle BAC$

Prove: $\overline{AM}$ is a median

Statements	Reasons
1) $\overline{AM}$ is an altitude	1) Given
2) $\triangle BMA$ & $\triangle CMA$ are Rt $\triangle$'s	2) Alt forms Rt $\angle$'s in $\triangle$
3) $\angle BMA \cong \angle CMA$	3) All right $\angle$'s are $\cong$
4) $\overrightarrow{AM}$ bis $\angle BAC$	4) Given
5) $\angle BAM \cong \angle CAM$	5) A bis. $\angle$ is $\div$ into $2 \cong \angle$'s
6) $\overline{AM} \cong \overline{AM}$	6) Reflexive Prop.
7) $\triangle BAM \cong \triangle CAM$	7) ASA
8) $\overline{BM} \cong \overline{MC}$	8) CPCTC
9) $\overline{AM}$ is a median	9) If $\triangle$ side $\div$ into $2 \cong$ segs $\rightarrow$ median

3.



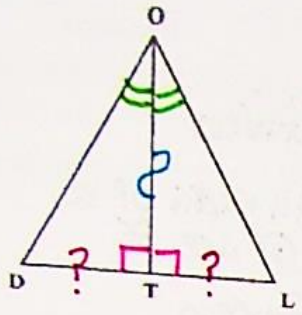
Given: $\overline{AM}$ is a median, $\overline{AB} \cong \overline{AC}$

Prove: $\overrightarrow{AM}$ bis. $\angle BAC$

Statements	Reasons
1) $\overline{AM}$ is a median	1) Given
2) $\overline{BM} \cong \overline{MC}$	2) med $\div$ opp side of $\triangle$ into $2 \cong$ segs
3) $\overline{AB} \cong \overline{AC}$	3) Given
4) $\overline{AM} \cong \overline{AM}$	4) Reflexive Prop.
5) $\triangle ABM \cong \triangle ACM$	5) SSS
6) $\angle BAM \cong \angle CAM$	6) CPCTC
7) $\overrightarrow{AM}$ bis $\angle BAC$	7) If $\angle$ is $\div$ into $2 \cong \angle$'s, then bis by a ray.

Beyond CPCTC (OTB 3.4)

1)



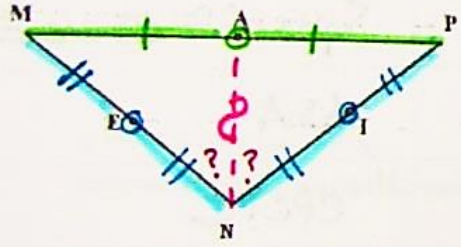
Given: $\overline{OT}$ is an altitude

$\overline{OT}$ bisects ∠DOL

Prove: $\overline{OT}$ is a median

Statements	Reasons
1) $\overline{OT}$ is an alt	1) Given
2) ∠OTD & ∠OTL are Rt ∠'s	2) Δ alt forms Rt ∠s
3) ∠OTD ≅ ∠OTL	3) All right ∠s are ≅
4) $\overline{OT}$ bis ∠DOL	4) Given
5) ∠DOT ≅ ∠LOT	5) A bis ∠ is ÷ into 2 ≅ ∠s
6) $\overline{OT} \cong \overline{OT}$	6) Reflexive Prop.
7) ΔDOT ≅ ΔLOT	7) ASA
8) $\overline{DT} \cong \overline{TL}$	8) CPCTC
9) $\overline{OT}$ is a median	9) If opp side of Δ is ÷ into 2 ≅ segs, then median

2)

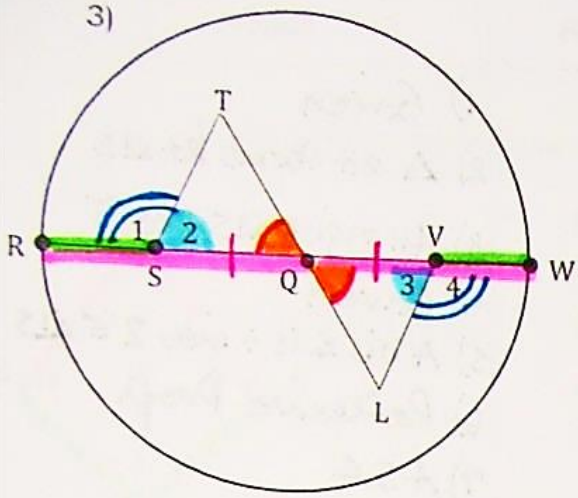


Given: I midpt $\overline{PN}$
E midpt $\overline{MN}$
$\overline{EN} \cong \overline{IN}$
A midpt $\overline{PM}$

Prove: $\overline{NA}$ bisects ∠MNP

Statements	Reasons
1) I mdpt of $\overline{PN}$	1) Given
2) E mdpt of $\overline{MN}$	2) Given
3) $\overline{EN} \cong \overline{IN}$	3) Given
4) $\overline{NM} \cong \overline{NP}$	4) Multiplication Property
5) A mdpt of $\overline{PM}$	5) Given
6) $\overline{MA} \cong \overline{AP}$	6) mdpt ÷ seg into 2 ≅ segs
7) Draw $\overline{NA}$	7) 2 pts det. a seg.
8) $\overline{NA} \cong \overline{NA}$	8) Reflexive Prop.
9) ΔMAN ≅ ΔPAN	9) SSS
10) ∠MNA ≅ ∠PNA	10) CPCTC
11) $\overrightarrow{NA}$ bis. ∠MNP	11) If ∠ is ÷ into 2 ≅ ∠s, then bis. by a ray

3)



Given: $\odot Q$
$\overline{RS} \cong \overline{WV}$
$\measuredangle 1 \cong \measuredangle 4$
Prove: Q midpt of $\overline{TL}$

Statements	Reasons
1. $\odot Q$	1. Given
2. $\overline{QR} \cong QW$	2. All radii of a $\odot$ are $\cong$
3. $\overline{RS} \cong \overline{WV}$	3. Given
4. $\overline{SQ} \cong \overline{QV}$	4. Subtraction Prop.
5. $\measuredangle SQT$ & $\measuredangle VQL$ are vertical $\measuredangle$'s	5. Assumed from diagram
6. $\measuredangle SQT \cong \measuredangle VQL$	6. Vertical $\measuredangle$'s are $\cong$
7. $\measuredangle 1 \cong \measuredangle 4$	7. Given
8. $\measuredangle 1$ supps $\measuredangle 2$ $\measuredangle 4$ supps $\measuredangle 3$	8. If 2 $\measuredangle$'s form str $\measuredangle$, then supp
9. $\measuredangle 2 \cong \measuredangle 3$	9. Supps of $\cong$ $\measuredangle$'s are $\cong$
10. $\Delta SQT \cong \Delta VQL$	10. ASA
11. $\overline{TQ} \cong \overline{QL}$	11. CPCTC
12. Q mdpt of $\overline{TL}$	12. If seg is $\div$ into 2 $\cong$ segs by a pt, then midpoint