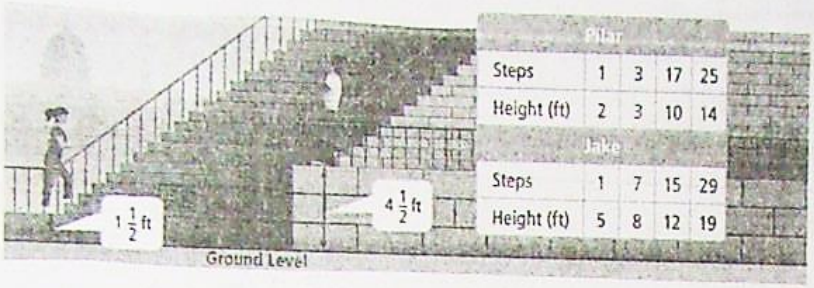
Topic 2-4 "Parallel & Perpendicular Lines"

Activity

MODEL & DISCUSS

Pilar and Jake begin climbing to the top of a 100-ft monument at the same time along two different sets of steps at the same rate. The tables show their distances above ground level after a number of steps.



Pilar				
Steps	1	3	17	25
Height (ft)	2	3	10	14

Jake				
Steps	1	7	15	29
Height (ft)	5	8	12	19

2-4 Slopes of Parallel and Perpendicular Lines

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Answer

$\frac{1 \text{ ft}}{2 \text{ steps}} = \frac{x \text{ ft}}{10 \text{ steps}}$

$2x = 10$

$x = 5$ ft

A. How many feet does each student climb after 10 steps? Explain.

Pilar

Steps	Height
1	2
3	3 (+1)
17 (+14)	10 (+7)
25 (+8)	14 (+4)

$\frac{\text{height}}{\text{steps}} = \frac{1}{2} = \frac{7}{14} = \frac{4}{8}$

1 foot every 2 steps

Jake

Steps	Height
1	5
7 (+6)	8 (+3)
15 (+8)	12 (+4)
29 (+14)	19 (+7)

$\frac{\text{height}}{\text{steps}} = \frac{3}{6} = \frac{4}{8} = \frac{7}{14} = \frac{1}{2}$

1 foot every 2 steps

B. Will Pilar and Jake be at the same height after the same number of steps? Explain.

No. Pilar starts at $1\frac{1}{2}$ ft above ground level and Jake starts at $4\frac{1}{2}$ ft. Since they both gain one foot of height every two steps, Jake will always be above Pilar's level off the ground.

C. Reason What would you expect the graphs of each to look like given your answers to parts A and B? Explain. MP.2

A line graphed for each person would result in a pair of parallel lines.

HABITS OF MIND

Model With Mathematics What would it mean if the graphs intersected? MP.4

If the graphs intersected, the point of intersection would be the number of steps paired with a height where they would be the same.

Assess

EXAMPLE 1 **Try It! Slopes of Parallel Lines**

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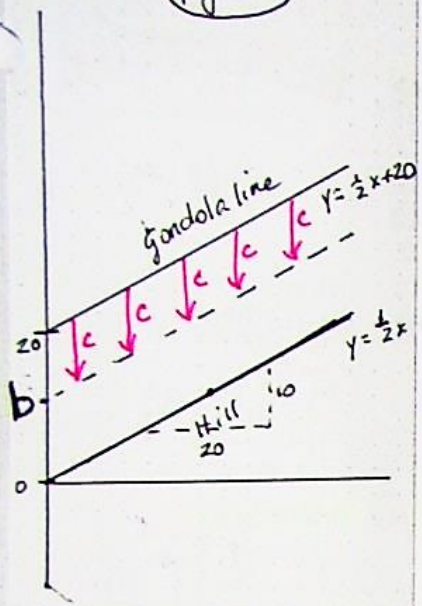

1. Suppose another line for a chair lift is placed at a constant distance *c* below the gondola line. What is an equation of the new line? Is the new line also parallel to the hill? Explain.

$y = mx + b$

Equation for new line: $y = \frac{1}{2}x + c$

$(c < 20)$

~ or ~ $y = \frac{1}{2}x + (20 - c)$ "b"

EXAMPLE 2 **Try It! Check Parallelism**

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2. Are lines *m* and *q* parallel?

slope of $m = \frac{1}{3}$ $(-4, 3)(-1, 4)$

slope of $q = \frac{2}{5}$ $(-4, -2)(1, 0)$

$\frac{1}{3} \neq \frac{2}{5}$ $m \nparallel q$

No

HABITS OF MIND

Reason Could two lines that are parallel ever pass through the same point? MP.2

No. Only if the two lines are the same line could they pass through the same point

Assess

EXAMPLE 3 **Try It! Check Perpendicularity**

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3. a. Are lines h and ℓ perpendicular?

$(1,-3)\ (-2,-1)$ slope of h: $\frac{-3-(-1)}{1-(-2)} = \frac{-3+1}{1+2} = -\frac{2}{3}$

$(3,2)\ (1,-3)$ slope of ℓ: $\frac{2-(-3)}{3-1} = \frac{2+3}{2} = \frac{5}{2}$

$\left(-\frac{2}{3}\right)\left(\frac{5}{2}\right) = -\frac{5}{3}$

$-\frac{5}{3} \neq -1$ No, not $\perp$

b. Are lines k and m perpendicular?

$(-3,2)\ (0,4)$ slope of k: $\frac{4-2}{0-(-3)} = \frac{2}{3}$

$(0,3)\ (2,-5)$ slope of m: $\frac{-5-3}{2-0} = \frac{-8}{2} = -4$

$\left(\frac{2}{3}\right)\left(\frac{-4}{1}\right) = -\frac{8}{3}$

$-\frac{8}{3} \neq -1$ No, not $\perp$

EXAMPLE 4 **Try It! Write Equations of Parallel and Perpendicular Lines**

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4. What are equations of lines parallel and perpendicular to the given line k passing through point T?

a. $y = -3x + 2$; $T(3, 1)$

Find b $x=3$ $y=1$ $m=-3$

$y_{\parallel} = mx+b$

$1 = -3(3)+b$

$1 = -9+b$

$10 = b$

Parallel Eq: $y = -3x+10$

$y_{\perp} = \frac{1}{3}x+b$ $x=3$ $y=1$ $m=\frac{1}{3}$

$1 = \frac{1}{3}(3)+b$

$1 = 1+b$

$0 = b$

Perpendicular Eq: $y = \frac{1}{3}x$

b. $y = \frac{3}{4}x - 5$; $T(12, -2)$

$y_{\parallel} = mx+b$ $y=-2$ $x=12$ $m=\frac{3}{4}$

$-2 = \frac{3}{4}(12)+b$

$-2 = 9+b$

$-11 = b$

Parallel Eq: $y = \frac{3}{4}x - 11$

$y_{\perp} = mx+b$ $y=-2$ $x=12$ $m=-\frac{4}{3}$

$-2 = -\frac{4}{3}(12)+b$

$-2 = -16+b$

$14 = b$

Perpendicular Eq: $y = -\frac{4}{3}x+14$

HABITS OF MIND

Generalize How can you use slope to determine if two lines are perpendicular, parallel, or neither perpendicular or parallel? MP.8

- Parallel lines have the same slope & diff y-intercepts
- Perpendicular lines have opposite reciprocal slopes (the product of the slopes is -1)
- Neither — the slopes are different and do not have a product of -1

Assess

Do You UNDERSTAND?

1. ESSENTIAL QUESTION How do the slopes of the lines that are parallel to each other compare? How do the slopes of the lines that are perpendicular to each other compare?

(∥) Parallel – Same slope

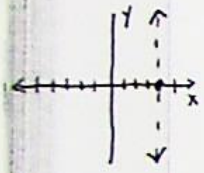
(⊥) Perpendicular – opposite reciprocal slopes

$(m_1)(m_2) = -1$

2. **Error Analysis** Katrina said that the lines $y = -\frac{2}{3}x + 5$ and $y = -\frac{3}{2}x + 2$ are perpendicular. Explain Katrina's error. MP.3

The product of two negative numbers is positive; in this case $(-\frac{2}{3})(-\frac{3}{2}) = 1$. The product of the slopes of ⊥ lines is -1

3. **Reason** Give an equation for a line perpendicular to the line $y = 0$. Is there more than one such line? Explain. MP.2



$x = 4$ yes, any vertical line is perpendicular to $y = 0$ (the line) i.e. (the x-axis), so $x = -4$, $x = 2$, $x = -10$ etc

4. **Communicate Precisely** What are two different if-then statements implied by Theorem 2-13? MP.6

Thm 2-13: Two non-vertical lines are parallel if and only if their slopes are equal.

- If the slopes of two non-vertical lines are equal, then they are parallel
- If two non-vertical lines are parallel, then their slopes are equal

5. **Error Analysis** Devin said that $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ for $A(-2, 0)$, $B(2, 3)$, $C(1, -1)$, and $D(5, -4)$ are parallel. Explain and correct Devin's error. MP.3

$A(-2,0)\ B(2,3)$ slope of $\overleftrightarrow{AB}$: $\frac{3-0}{2-(-2)} = \frac{3}{4}$

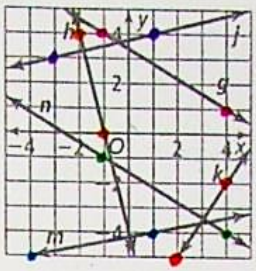
$C(1,-1)\ D(5,-4)$ slope of $\overleftrightarrow{CD}$: $\frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-(-4)}{5-1} = \frac{3}{4}$

$(x_1, y_1)\ (x_2, y_2)$ slopes are equal, so $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ ✗

Devin did not start with the same point when calculating $\frac{\Delta y}{\Delta x}$ for line CD.

Do You KNOW HOW?

Use the diagram for Exercises 6–9.



6. Are lines g and n parallel?

$g(-1,4)(4,1)$ $m = \frac{1-4}{4-(-1)} = \frac{-3}{4+1} = -\frac{3}{5}$

$n(-1,-1)(4,-4)$ $m = \frac{-4-(-1)}{4-(-1)} = \frac{-4+1}{4+1} = -\frac{3}{5}$

$-\frac{3}{5} = -\frac{3}{5}$ Yes

7. Are lines j and m parallel?

$j(-3,3)(1,4)$ $m = \frac{4-3}{1-(-3)} = \frac{1}{1+3} = \frac{1}{4}$

$m(-4,-5)(1,-4)$ $m = \frac{-4-(-5)}{1-(-4)} = \frac{-4+5}{1+4} = \frac{1}{5}$

$\frac{1}{4} \neq \frac{1}{5}$ No

8. Are lines n and k perpendicular?

$n(-1,-1)(4,-4)$ $m = \frac{-4-(-1)}{4-(-1)} = \frac{-4+1}{4+1} = -\frac{3}{5}$

$k(2,-5)(4,-2)$ $m = \frac{-2-(-5)}{4-2} = \frac{-2+5}{2} = \frac{3}{2}$

$(-\frac{3}{5})(\frac{3}{2}) \neq -1$ No

9. Are lines h and j perpendicular?

$h(-2,4)(-1,0)$ $m = \frac{0-4}{-1-(-2)} = \frac{-4}{-1+2} = \frac{-4}{1} = -4$

$j(-3,3)(1,4)$ $m = \frac{4-3}{1-(-3)} = \frac{1}{1+3} = \frac{1}{4}$

$(-4)(\frac{1}{4}) = -1$ Yes

10. What is an equation for the line parallel to $y = -x + 7$ that passes through $(7, -2)$?

$m = -1$ $x = 7$ $y = -2$

Find b: $-2 = -1(7) + b$, $-2 = -7 + b$, $5 = b$

$y = -x + 5$

11. What is an equation for the line perpendicular to $y = 3x - 1$ that passes through $(-9, -2)$?

$m = -\frac{1}{3}$ $x = -9$ $y = -2$

Find b: $-2 = -\frac{1}{3}(-9) + b$, $-2 = 3 + b$, $-5 = b$

$y = -\frac{1}{3}x - 5$

12. The graph of a roller coaster track goes in a straight line through coordinates (10, 54) and (42, 48), with coordinates in feet. A support beam runs parallel 12 feet below the track. What equation describes the support beam?

slope $= \frac{54-48}{10-42} = \frac{6}{-32} = -\frac{3}{16} = m$

$x = 10$ $y = 54$ Find b: $54 = -\frac{3}{16}(10) + b$, $54 = -\frac{30}{16} + b$, $54 + 1\frac{7}{8} = b$

$b = 55\frac{7}{8}$

Subtract 12 → $b = 43\frac{7}{8}$

Equation: $y = -\frac{3}{16}x + 43\frac{7}{8}$

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