

Topic 1-8 Proof by Contrapositive

(also see examples in **GREEN** Text Book)

1 - 8 Indirect Proofs

thought bubbles pg 60

EXAMPLE 3

Write an indirect proof of the following statement using proof by **CONTRAPOSITIVE**.

For two positive integers n and m , if $nm > 16$, then either n or m is greater than 4 or both are greater than 4.

If the product of n times m is not greater than 16, then it must be less than or equal to 16.

If both m and n are not greater than 4, then one or both must be less than or equal to 4.

Step 1: Identify p and q , and then write the negation of p and q .

Let $p = nm > 16$

Let $q = m > 4 \quad n > 4$

Let $\sim p = nm \leq 16$

$\sim q \rightarrow \sim p$

Let $\sim q = m \leq 4 \quad n \leq 4$

Step 2: Assume $\sim q$ is true.

Suppose: $m \leq 4 \quad \text{or} \quad n \leq 4$

Step 3: Show that your assumption from step 2 leads to $\sim p$.

If $n \leq 4$

Let $n = 4$

Then, $nm \leq 4m$

$nm \leq 4m$

If $m \leq 4$

Let $m = 4$

Then, $4m \leq 16$

$4m \leq 16$

By the **transitive property**, $nm \leq 16$

transitive property means

by comparison

Ex: $\triangle A \cong \triangle B$
 $\triangle C \cong \triangle B$
 $\therefore \triangle A \cong \triangle C$

(Therefore) $\sim q \rightarrow \sim p$, therefore $p \rightarrow q$ and

(If $nm > 16$, then $n > 4$ and $m > 4$ or both)

Step 4: Conclude

TRUE
Contrapositive If $n \leq 4$ and $m \leq 4$ or both are ≤ 4 , then $nm \leq 16$

Ex Prove by contraposition:

Given: $BS = 56$

Prove: $x \neq 6$

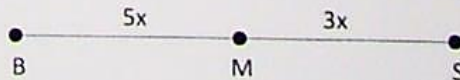
$p: BS = 56$

$q: x \neq 6$

$\sim p: BS \neq 56$

$\sim q: x = 6$

($\sim q \rightarrow \sim p$)



If $x = 6$, then $BS = 5x + 3x = 8x$.
 Now $BS = 8(6) = 48$. Since $48 \neq 56$, the contrapositive $\sim q \rightarrow \sim p$ is true and $x \neq 6$.
 \therefore If $BS = 56$, then $x \neq 6$