

(also see examples in **GREEN** Text Book)






## 1-5

### Conditional Statements

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#### EXPLORE & REASON

If-then statements show a cause and effect. The table shows some if-then statements.

Cause	Effect
1) If it is raining 	then it is spring. 
2) If $x$ and $y$ are whole numbers	then their difference, $x - y$ , is a whole number.
3) If water is heated 	then it boils. 
4) If a triangle has a right angle	then it is a right triangle.
5) If your favorite color is blue 	then you are a good speller.

A. **Construct Arguments** Determine whether each effect is always true for the given cause, or is not necessarily true for the given cause. For the effects that are not necessarily true, how could you change them to make them always true? **MP.3**

The fourth statement is the only one that is always true. The other statements need the following changes to be true:

- 1) If it is raining between winter and summer, then it is spring
- 2) Change the term whole numbers to integers
- 3) If water is heated to  $100^{\circ}\text{C}$ , then it boils
- 5) If your favorite color is blue, then your favorite color is not green

B. Write some if-then statements of your own. Write two statements that are always true and two statements that are not necessarily true.

Always true:

If it rains, then the house gets wet.

If today is Thursday, then tomorrow is Friday.

Not necessarily true:

If you like to eat salads, then you are a vegetarian

If you like to swim, then you like to jog.

#### HABITS OF MIND

**Look for Relationships** You have used the word *hypothesis* and *conclusion* in other classes. How are their meanings the same as or different from the definitions in geometry? **MP.7**

In science classes, a hypothesis is a conjecture based on observation. This meaning differs from that used in formal logic.

In geometry, a hypothesis is the cause in a cause-and-effect statement.

1.5

## EXAMPLE 1

p. 37

## Try It! Write a Conditional Statement

1. Write each statement as a conditional.

a. A triangle with all angles congruent is equilateral.

- If a triangle has all angles congruent, then it is an equilateral triangle
- or - If a triangle is equilateral, then it has all angles congruent.

b. Alberto can go to the movies if he washes the car.

If Alberto washes the car, then he can go to the movies.

## EXAMPLE 2

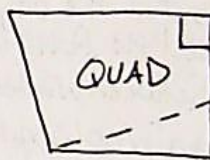
p. 37

## Try It! Find a Truth Value of a Conditional

2. What is the truth value of each conditional? Explain your reasoning.

a. If a quadrilateral has a right angle, then it is a rectangle.

False



or trapezoid

b. If  $X$  is the midpoint of  $\overline{AB}$ , then  $X$  lies on  $\overline{AB}$ .

True By definition of a midpoint, the point must be a point on the segment to divide it into 2  $\cong$  segments

## EXAMPLE 3

p. 38

## Try It! Write and Determine the Truth Value of a Converse

3. Write and determine the truth value of the converse of the conditional.

a. If a polygon is a quadrilateral, then it has four sides.

If a polygon has four sides, then it is a quadrilateral, True

b. If two angles are complementary, then their angle measures add to  $90^\circ$ .

If the measure of two angles adds to  $90^\circ$ , then they are complementary.

True

1.5

**EXAMPLE 4** **Try It!** Write and Evaluate the Truth Value of an Inverse and a Contrapositive

p. 39

(F)  $p \rightarrow q$

(T)  $q \rightarrow p$

(T)  $\sim p \rightarrow \sim q$

(F)  $\sim q \rightarrow \sim p$

4. Write the converse, the inverse, and the contrapositive. What is the truth value of each?

If today is a weekend day, then tomorrow is Monday.

If tomorrow is Monday, then today is a weekend day

If today is not a weekend day, then tomorrow is not Monday

If tomorrow is not Monday, then today is not a weekend day

**HABITS OF MIND****Reason** What is the truth value of the conditional, "If 11 is an even number, then there are 23 hours in a day?" What are the truth values of the hypothesis and conclusion? Is this true of all conditionals where the hypothesis can never be true? **MP2**

The statement is true since both the hypothesis &amp; conclusion are false. A conditional statement is considered true by definition if the hypothesis is false. Unlike in non sense statements, this concept is easier to understand if the hypothesis actually leads to the conclusion. Since the conclusion can be true or false, both conditionals are true at least some of the time.

**EXAMPLE 5** **Try It!** Write and Evaluate a Biconditional

p. 40

5. Write a biconditional for the following conditional. What is its truth value?

If two lines intersect at right angles, then they are perpendicular.

Two lines intersect at right angles if and only if they are perpendicular.

True

 $p$  = two lines intersect at right angles $q$  = two lines are perpendicular**EXAMPLE 6** **Try It!** Identify the Conditionals in a Biconditional

p. 40

6. What are the two conditionals implied by the biconditional?

The product of two numbers is negative if and only if the numbers have opposite signs.

- If the product of two numbers is negative, then the numbers have opposite signs
- If two numbers have opposite signs, then their product is negative.

**HABITS OF MIND****Generalize** How is a biconditional similar to giving a definition? Can you think of a definition in geometry and express it as a biconditional? **MP8**

A biconditional is like a definition because the converses of both are also true.

Ex: A quadrilateral is a square if and only if it is equiangular and equilateral.

Pg 41

**Do You UNDERSTAND?**

15

1. **ESSENTIAL QUESTION** How do if-then statements describe mathematical relationships.

Cause & effect  
(hypothesis) (conclusion)

2. **Error Analysis** Allie was asked to write the inverse of the following conditional.

If it is sunny, then I use sunscreen.

What error did Allie make? © MP3

✓  
If it is not sunny, then I use sunscreen.

X

$\sim P \rightarrow \sim q$   
Allie forgot to negate the conclusion, too.

3. **Vocabulary** Which term is used to describe the opposite of a statement?

Negation ( $\sim$ ) "not"

4. **Generalize** How do you write the converse of a conditional? How do you write the contrapositive of a conditional? © MP8

- Conditional  $P \rightarrow q$
- Converse  $q \rightarrow P$
- Inverse  $\sim P \rightarrow \sim q$
- Contrapositive  $\sim q \rightarrow \sim P$

5. **Communicate Precisely** Explain how the inverse and the contrapositive of a conditional are alike and how they are different. © MP6

Both: hypothesis & conclusion are negated  
Contrapositive: hypothesis & conclusion of conditional are reversed

**Do You KNOW HOW?**

6. Write the following statement as a biconditional.

A prime number has only 1 and itself as factors.

A number is prime if and only if it has only 1 and itself as factors.

For Exercises 7-9, use the following conditional.

If a rectangle has an area of  $12 \text{ m}^2$ , then it has sides of length 3 m and 4 m.

7. What is the hypothesis? What is the conclusion?

P: If a rectangle has an area of  $12 \text{ m}^2$   
q: then it has sides of length 3m & 4m

8. Assume the hypothesis is false. What is the truth value of the conditional? Assume the hypothesis is true. What would be a counterexample?

True  
A rectangle with sides of length 2m and 6m

9. What are the converse, the inverse, and the contrapositive? What are their truth values?

- $q \rightarrow P$
- $\sim P \rightarrow \sim q$
- $\sim q \rightarrow \sim P$

10. What two conditionals are implied by the following biconditional?

"The city can build new roads if and only if the sales tax is raised to 10%."



If a city can build new roads, then it raised the sales tax to 10%.  
If the city raised the sales tax to 10%, then it can build new roads.