1.7 Deductive Structure

Lesson Objective *After studying this section, you will be able to:*

- *Recognize that geometry is based on a deductive structure*
- *Identify undefined terms, postulates, and definitions*
- *Understand the characteristics of theorems and the ways in which they can be used in proofs*

Recognizing the difference between deductive reasoning and inductive reasoning:

Deductive Reasoning is a system of thought based on statements that have already been proven or accepted as fact.

Example: *If you add two odd numbers together, the solution is always even. John is given a problem of adding 111 and 47 together. He concludes that his answer will be even.*

Inductive Reasoning is a system of thought based on observation.

Example: *Peter uses his calculator to add 3 + 5 and gets 8. He repeats the process of adding two odd numbers and continues to get even number sums. He concludes that every time two odd numbers are added together, the sum is even.*

The Four Parts of the Geometric Deductive System are:

1. Undefined terms - point, line, and plane
2. Definitions- the meaning of a term or idea
3. Postulates- statements accepted as true without any proof (assumptions)
4. Theorems- statements that are proven

Declarative Statements… (I declare… "If p, then q!")

- All definitions, postulates and theorems may be written in "if ____ then____" form.
- The "if____, then ______" form of a statement is called a **conditional statement.**
- ***Conditional Statements*** are written in *"if ____ then ____"* form.
- The phrase following the "if ___" part of the statement is called the ***hypothesis***.
- The phrase following the *"then ____"* part of the statement is called the ***conclusion***.

*The **hypothesis** contains the given information and the **conclusion** is what must be proved.*

Conditional Statement: *If you study, you will do well on your geometry exams!*

hypothesis: *"you study"* ***conclusion:*** *"you do well on exams"*

Converse Statement: When the hypothesis and the conclusion are exchanged.

Declarative Statement: "If p, then q"

Converse: "If q, then p"

Example 1: *(If p, then q)* ***If** you study, **then** you will do well on your geometry exams.*

(If q, then p) If you did well on your geometry exam, then you studied.

Example 2: Given the conditional, *"If p, then q":*

If two angles are both right angles then they are congruent.

***Hypothesis** ("If p"):* Two angles are both right angles

***Conclusion** ("then q"):* They are congruent.

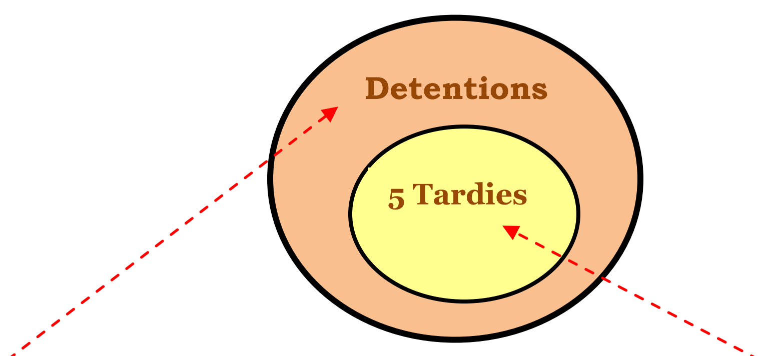
The converse is ("If q, then p"): If two angles are congruent then they are both right angles.

Note: The converse is ***false!*** Two angles of 30 degrees each are congruent but NOT right.

Counterexample: A counterexample is when you give an example to prove a statement is incorrect or false, as was the case above.

To reason from a conditional, often a Venn diagram is made.

Example: If a student is tardy 5 times then the student will be given a detention.



The **conclusion** is always in the **big circle,** while the **hypothesis** in the **small circle**.

If there is only ONE place to put the person, **then** a conclusion can be made.

- John has 5 tardies. Conclusion: He has a detention.
- Mary has a detention. No conclusion! It could be that Mary received a detention for 5 tardies or for some other reason.
- Linda does not have a detention. Conclusion: She does not have 5 tardies.
- Peter does not have 5 tardies. No conclusion! He may or may not have a detention!