NAME ______________________ DATE ______

Student Edition
Pages 672–675

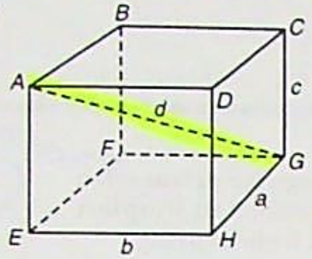
13-3 Enrichment

9.8 Diagonals

To find the length of diagonals in cubes and rectangular solids, a formula can be applied. In the example below, the length of diagonal $\overline{AG}$ or d can be found using the formula

$$d^2 = a^2 + b^2 + c^2 \text{ or } d = \sqrt{a^2 + b^2 + c^2}. \quad \text{or } d = \sqrt{\ell^2 + w^2 + h^2}$$

Example:



The diagonal, d, is equal to the square root of the sum of the squares of the length, a, the width, b, and the height, c.

Example: Find the length of the diagonal of a rectangular prism with length of 8 meters, width of 6 meters, and height of 10 meters.

$d = \sqrt{8^2 + 6^2 + 10^2}$ Substitute the dimensions into the equation.

$= \sqrt{64 + 36 + 100}$ Square each value. Add.

$= \sqrt{200} < {}^{100}_{2}$ Find the square root of the sum.

~~≈ 14.1 m~~ ~~Round the answer to the nearest tenth.~~

$= 10\sqrt{2}$ ← Write in simplest radical form!

Solve. Use $d = \sqrt{a^2 + b^2 + c^2}$. Round answers to the nearest tenth.

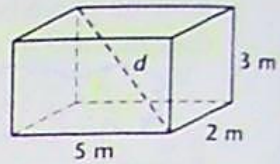
1. Find the diagonal of a cube with sides of 6 inches.
   $d = \sqrt{6^2 + 6^2 + 6^2} = \sqrt{3 \cdot 36} = \boxed{6\sqrt{3} \text{ in}}$
2. Find the diagonal of a cube with sides of 2.4 meters.
   $d = \sqrt{(2.4)^2 + (2.4)^2 + (2.4)^2} = \sqrt{3 \cdot (2.4)^2} = \boxed{2.4\sqrt{3} \text{ m}}$
3. Find the diagonal of a rectangular solid with length of 18 meters, width of 16 meters, and height of 24 meters.
   $d = \sqrt{18^2 + 16^2 + 24^2} = \sqrt{324 + 256 + 576} = \sqrt{1156} = \boxed{34 \text{ m}}$
4. Find the diagonal of a rectangular solid with length of 15.1 meters, width of 8.4 meters, and height of 6.3 meters.
   $d = \sqrt{(15.1)^2 + (8.4)^2 + (6.3)^2} = \sqrt{228.01 + 70.56 + 39.69} = \boxed{\sqrt{338.26} \text{ m}}$ or $\boxed{\approx 18.4 \text{ m}}$
5. Find the diagonal of a cube with sides of 34 millimeters.
   $d = \sqrt{(34)^2 + (34)^2 + (34)^2} = \sqrt{3 \cdot (34)^2} = \boxed{34\sqrt{3} \text{ mm}}$
6. Find the diagonal of a rectangular solid with length of 8.9 millimeters, width of 6.7 millimeters, and height of 14 millimeters.
   $d = \sqrt{(8.9)^2 + (6.7)^2 + (14)^2} = \sqrt{79.21 + 44.89 + 196} = \boxed{\sqrt{320.1} \text{ mm}}$ or $\boxed{\approx 17.9 \text{ mm}}$

◆ Skill B Using the formula for the length of a diagonal of a right rectangular prism

Recall A diagonal of a polyhedron is a segment that joins two points that are vertices of different faces of the polyhedron. The Pythagorean Theorem can be used to derive a formula for the length of a diagonal of a right rectangular prism.

If a right rectangular prism has length ℓ, width w, and height h, then the length, d, of a diagonal is given by $d = \sqrt{\ell^2 + w^2 + h^2}$.

◆ Example
Find the length of a diagonal of the right rectangular prism.



◆ Solution
$d = \sqrt{\ell^2 + w^2 + h^2} = \sqrt{5^2 + 2^3 + 3^2} = \sqrt{38} \approx 6.16$ m

Find the length of a diagonal of a right rectangular prism with the given dimensions. Give your answer as a radical in simplest form and as a decimal rounded to the nearest hundredth. Work shown below

10. $\ell = 9, w = 4, h = 5$ $\sqrt{122} \approx 11.05$

11. $\ell = 2, w = 2, h = 4$ $\sqrt{24} = 2\sqrt{6} \approx 4.90$

12. $\ell = 3, w = 7, h = 2$ $\sqrt{62} \approx 7.87$

13. $\ell = 3, w = 2, h = 3$ $\sqrt{22} \approx 4.69$

14. $\ell = 15, w = 6, h = 9$ $\sqrt{342} = 3\sqrt{38} \approx 18.49$

15. $\ell = 6, w = 12, h = 10$ $\sqrt{280} = 2\sqrt{70} \approx 16.73$

16. $\ell = 8, w = 10, h = 8$ $\sqrt{228} = 2\sqrt{57} \approx 15.10$

17. $\ell = 10, w = 8, h = 4$ $\sqrt{180} = 6\sqrt{5} \approx 13.42$

Find the missing dimension of the right rectangular prism. Give your answer as a radical in simplest form and as a decimal rounded to the nearest hundredth. Given the diagonal! Work shown below

18. $d = 17$, $w = 9$, $\ell = 12$, $h =$ 8

19. $d = 12$, $w = 8$, $h = 5$, $\ell =$ $\sqrt{55} \approx 7.42$

(10) $d = \sqrt{9^2+4^2+5^2}$
$= \sqrt{81+16+25}$
$\boxed{= \sqrt{122} \approx 11.05}$

(11) $d = \sqrt{2^2+2^2+4^2}$
$= \sqrt{4+4+16}$
$= \sqrt{24}$ <4, 6
$\boxed{= 2\sqrt{6} \approx 4.90}$

(12) $d = \sqrt{3^2+7^2+2^2}$
$d = \sqrt{9+49+4}$
$d \boxed{= \sqrt{62} \approx 7.87}$

(13) $d = \sqrt{3^2+2^2+3^2}$
$= \sqrt{9+4+9}$
$\boxed{= \sqrt{22} \approx 4.69}$

(14) $d = \sqrt{15^2+6^2+9^2}$
$= \sqrt{225+36+81}$
$= \sqrt{342}$ <9, 38
$\boxed{= 3\sqrt{38} \approx 18.49}$

(15) $d = \sqrt{6^2+12^2+10^2}$
$= \sqrt{36+144+100}$
$= \sqrt{280}$ <4, 70
$\boxed{= 2\sqrt{70} \approx 16.73}$

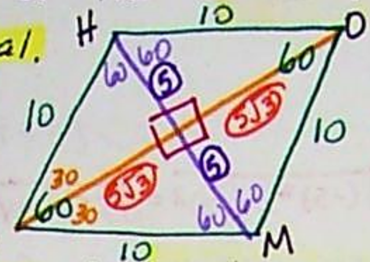
(16) $d = \sqrt{8^2+10^2+8^2}$
$= \sqrt{64+100+64}$
$= \sqrt{228}$ <4, 57
$\boxed{= 2\sqrt{57} \approx 15.10}$

(17) $d = \sqrt{10^2+8^2+4^2}$
$= \sqrt{100+64+16}$
$= \sqrt{180}$ <36, 5
$\boxed{= 6\sqrt{5} \approx 13.42}$

18
$17^2 = 9^2 + 12^2 + h^2$
$289 = 81 + 144 + h^2$
$289 = 225 + h^2$
$-225 \quad -225$
$\sqrt{64} = \sqrt{h^2}$
$\boxed{8 = h}$

19
$12 = \sqrt{8^2 + 5^2 + \ell^2}$
$12 = \sqrt{64 + 25 + \ell^2}$
$12 = \sqrt{89 + \ell^2}$
$(12)^2 = (\sqrt{89+\ell^2})^2$
$144 = 89 + \ell^2$
$-89 \quad -89$
$55 = \ell^2$
$\sqrt{\ell^2} = \sqrt{55}$
$\boxed{\ell = \sqrt{55} \approx 7.42}$

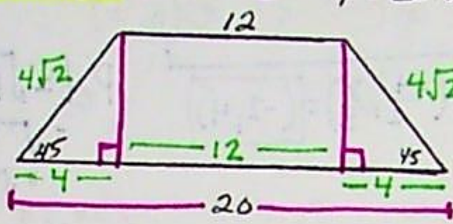
11. One of the angles of a rhombus has a measure of 60. If the perimeter of the rhombus is 40, find the length of each diagonal.



$RO = 2(5\sqrt{3}) = \boxed{10\sqrt{3}u}$

$HM = 2(5) = \boxed{10u}$

12. Find the perimeter of the trapezoid below.

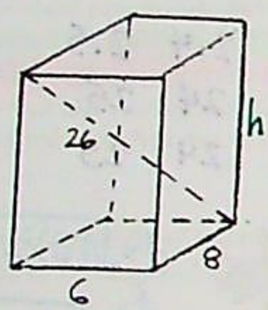


$P = 12 + 20 + 2(4\sqrt{2})$

$= \boxed{(32 + 8\sqrt{2})u}$

IV Spacial Figures

13.



$6^2 + 8^2 + h^2 = 26^2$

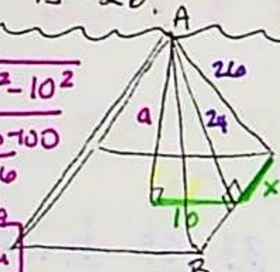
$36 + 64 + h^2 = 676$

$100 + h^2 = 676$

$\sqrt{h^2} = \sqrt{576}$

$\boxed{h = 24u}$

Find the height of the box whose base is 6 by 8 and whose diagonal is 26.



$a = \sqrt{24^2 - 10^2}$

$= \sqrt{576 - 100}$

$= \sqrt{476}$

$= \boxed{2\sqrt{119}u}$

regular square pyramid

Lateral edge = 26 = AB

Slant height = 24

Find altitude (height)

$\frac{x}{2} \quad \frac{24}{2} \quad \frac{26}{2} \qquad \frac{x}{2} = 5$

$5 \quad 12 \quad 13 \qquad \boxed{x = 10}$

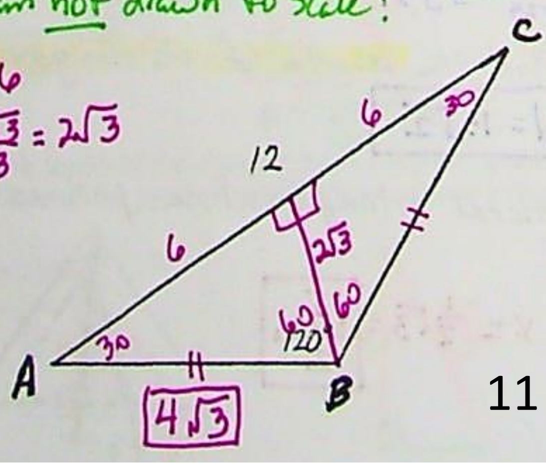
V Miscellaneous

14. Find AB in isos. △ABC

*Diagram not drawn to scale!

$x\sqrt{3} = 6$

$x = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$



15. Find the perimeter of the triangle. (hint: draw an altitude)

$P = 10 + 5 + 5\sqrt{2} + 5\sqrt{3}$

$= \boxed{(15 + 5\sqrt{2} + 5\sqrt{3})u}$