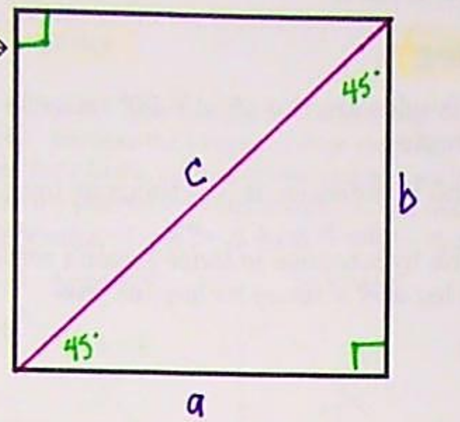
9.7 Special Right Triangles

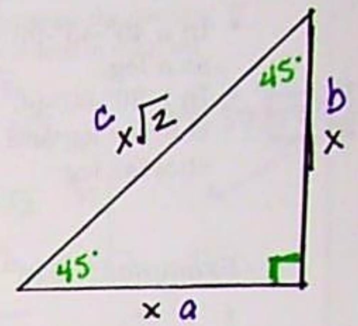
Part A) This is a *SQUARE* →



Directions:

- Use tick marks and any other marks that show its basic characteristics.
- Draw ONE diagonal
- Use the variables a, b, and c to label the sides of the two congruent triangles you formed.

- Draw one of the triangles below and label it with variables and any other measures that you know.



Follow-Up:

1) Write an expression showing the length of the diagonal:

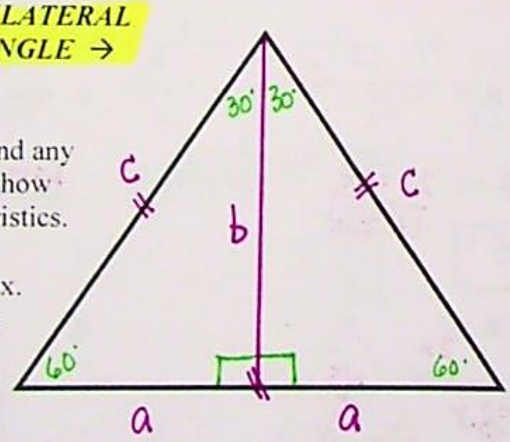
$c^2=a^2+b^2$ $c=\sqrt{a^2+b^2}$ $c=\sqrt{a^2+a^2}$ $c=\sqrt{2a^2}=a\sqrt{2}$

$c=a\sqrt{2}$ or $c=x\sqrt{2}$

2) Name the two sides of your triangle that have a special relationship. a & b What is it? a = b (x)

Write Theorem 73: In a 45-45-90° right triangle, the ratio of the sides always follow the ratio $x - x - x\sqrt{2}$, respectively. (i.e.: $1:1:\sqrt{2}$)

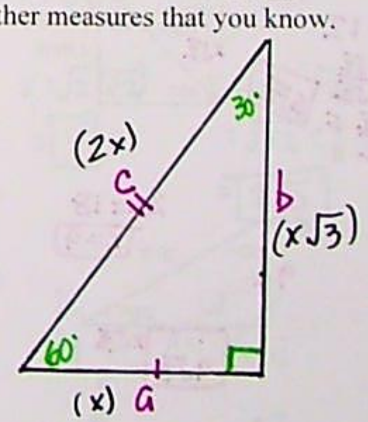
Part B) This is an *EQUILATERAL TRIANGLE* →



Directions:

- Use tick marks and any other marks that show its basic characteristics.
- Draw an altitude from the top vertex.
- Use the variables a, b, and c to label the sides of the two congruent triangles you formed.

- Draw one of the triangles below and label it with variables and any other measures that you know.



Follow-Up:

3) Write an expression showing the length of the altitude:

$b=\sqrt{c^2-a^2}=\sqrt{(2a)^2-(a)^2}=\sqrt{4a^2-a^2}=\sqrt{3c^2}=$

$b=a\sqrt{3}$ or $b=x\sqrt{3}$

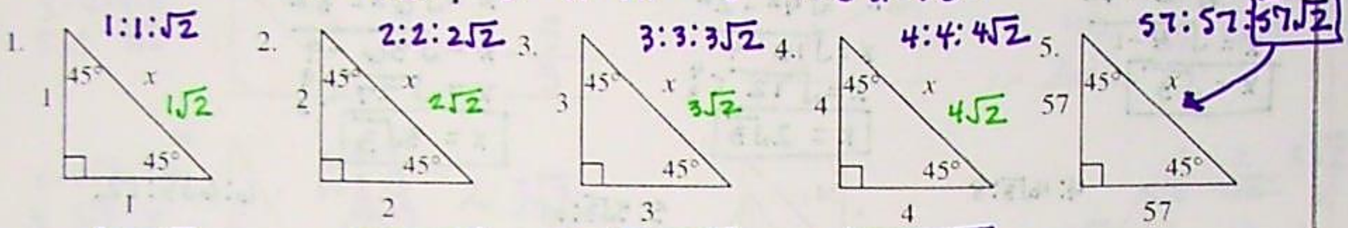
4) Name the two sides of your triangle that have a special relationship. a & c What is it? c = 2a

Write Theorem 72: In a 30-60-90° right triangle, the ratio of the sides always follows the ratio $x - x\sqrt{3} - 2x$, respectively. (i.e.: $1:\sqrt{3}:2$)

Name:______________________________________ Date:______________

Geometry: 9.7 Special Right Triangles

There are two right triangles which are looked at more than any others. They are the 45°-45°-90° and the 30°-60°-90°. They are "special" because the lengths of their sides follow a simple pattern. Use the Pythagorean Theorem to solve for x in the following triangles and see if you can determine the pattern. If you think you have figured out the pattern, then you should be able to do the **last** problem in each set without doing any work! Remember! $c^2 = a^2 + b^2 \Rightarrow c = \sqrt{a^2+b^2}$



1. $x = \sqrt{1^2+1^2}$, $x = \sqrt{1+1}$, $x = \sqrt{2}$

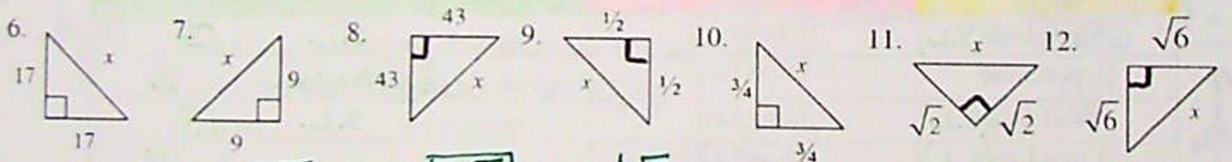
2. $x = \sqrt{2^2+2^2}$, $x = \sqrt{4+4}$, $x = \sqrt{2 \cdot 4}$, $x = 2\sqrt{2}$

3. $x = \sqrt{3^2+3^2}$, $x = \sqrt{9+9}$, $x = \sqrt{2 \cdot 9}$, $x = 3\sqrt{2}$

4. $x = \sqrt{4^2+4^2}$, $x = \sqrt{16+16}$, $x = \sqrt{2 \cdot 16}$, $x = 4\sqrt{2}$

Given	Find	Rule
Leg	Hypotenuse	*Multiply the* leg *by* $\sqrt{2}$
Hypotenuse	Leg	*Divide the* hypotenuse *by* $\sqrt{2}$

For questions 6-19, find the value of x in each of the isosceles right triangles.



6. $x = 17\sqrt{2}$

7. $x = 9\sqrt{2}$

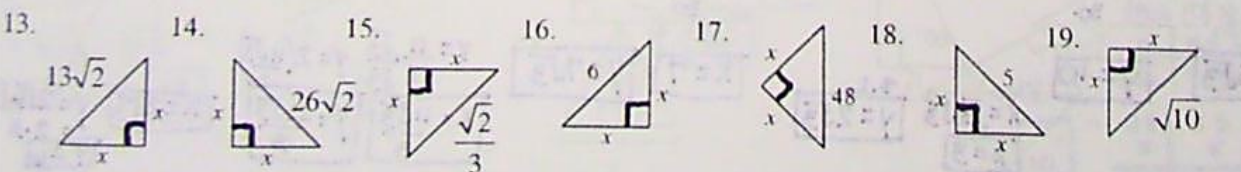
8. $x = 43\sqrt{2}$

9. $x = \frac{1}{2}\sqrt{2} = .5\sqrt{2} = \frac{\sqrt{2}}{2}$

10. $x = \frac{3}{4}\sqrt{2} = \frac{3\sqrt{2}}{4}$

11. $x = \sqrt{2}(\sqrt{2}) = 4$

12. $x = \sqrt{6}(\sqrt{2}) = \sqrt{12} \lessdot^{4}_{3} = 2\sqrt{3}$



13. $x = \frac{13\sqrt{2}}{\sqrt{2}}$, $x = 13$

14. $x = \frac{26\sqrt{2}}{\sqrt{2}}$, $x = 26$

15. $x = \frac{\frac{\sqrt{2}}{3}}{\frac{\sqrt{2}}{1}}$, $x = \frac{\sqrt{2}}{3} \cdot \frac{1}{\sqrt{2}}$, $x = \frac{1}{3}$

16. $x = \frac{6}{\sqrt{2}}$, $x = \frac{6\sqrt{2}}{2}$, $x = 3\sqrt{2}$

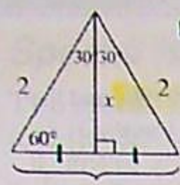
17. $x = \frac{48}{\sqrt{2}}$, $x = \frac{48\sqrt{2}}{2}$, $x = 24\sqrt{2}$

18. $x = \frac{5}{\sqrt{2}}$, $x = \frac{5\sqrt{2}}{2}$

19. $x = \frac{\sqrt{10}}{\sqrt{2}}$, $x = \frac{\sqrt{10}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$, $x = \frac{\sqrt{20}}{2} \lessdot^{4}_{5}$, $x = \frac{2\sqrt{5}}{2} = \sqrt{5}$

Remember! $a^2+b^2=c^2$, so $b=\sqrt{c^2-a^2}$

20. 1 : $\sqrt{3}$: 2

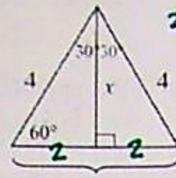


$x=\sqrt{2^2-1^2}$

$x=\sqrt{4-1}$

$x=\sqrt{3}$

21. 2 : $2\sqrt{3}$: 4



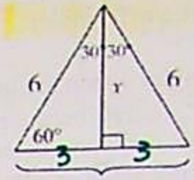
$x=\sqrt{4^2-2^2}$

$x=\sqrt{16-4}$

$x=\sqrt{12}$ <4, 3

$x=2\sqrt{3}$

22. 3 : $3\sqrt{3}$: 6



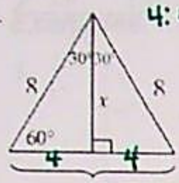
$x=\sqrt{6^2-3^2}$

$x=\sqrt{36-9}$

$x=\sqrt{27}$ <9, 3

$x=3\sqrt{3}$

23. 4 : $4\sqrt{3}$: 8



$x=\sqrt{8^2-4^2}$

$x=\sqrt{64-16}$

$x=\sqrt{48}$ <16, 3

$x=4\sqrt{3}$

24. 5 : $5\sqrt{3}$: 10

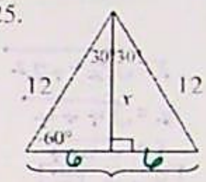
$x=\sqrt{10^2-5^2}$

$x=\sqrt{100-25}$

$x=\sqrt{75}$ <25, 3

$x=5\sqrt{3}$

25. 6 : $6\sqrt{3}$: 12

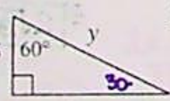


$x=6\sqrt{3}$

Given	Find	Rule
Short Leg (S.L.)	Hypotenuse	*Multiply the* S.L. *by* 2
Hypotenuse	Short Leg	*Divide the* hypotenuse *by* 2
Short Leg (S.L.)	Long Leg	*Multiply the* S.L. *by* $\sqrt{3}$
Long Leg (L.L.)	Short Leg	*Divide the* L.L. *by* $\sqrt{3}$

Find the values of x and y in the 30°-60°-90° triangles.

26.



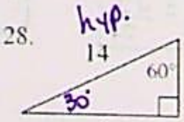
$x=5\sqrt{3}$ $y=10$

27.

$x=\sqrt{3}\cdot\sqrt{3}$

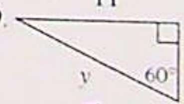
$x=3$ $y=2\sqrt{3}$

28.



$x=7$ $y=7\sqrt{3}$

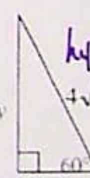
29.



$x=\frac{11}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}$ $y=2\left(\frac{11\sqrt{3}}{3}\right)$

$x=\frac{11\sqrt{3}}{3}$ $y=\frac{22\sqrt{3}}{3}$

30.

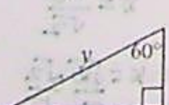


$x=2\sqrt{3}$ $y=2\sqrt{3}\sqrt{3}$

$y=2\cdot3$

$y=6$

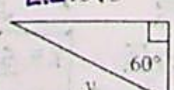
31.



$x=\frac{9\sqrt{3}}{\sqrt{3}}$ $y=18$

$x=9$

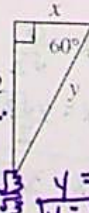
32.



$x=\frac{15\sqrt{3}}{\sqrt{3}}$ $y=2(15)$

$x=15$ $y=30$

33.

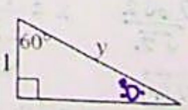


$x=\frac{12}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}$ $y=2x$

$x=\frac{12\sqrt{3}}{3}$ $y=8\sqrt{3}$

$x=4\sqrt{3}$

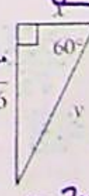
34.



$x=11\sqrt{3}$

$y=22$

35.



$x=\frac{2\sqrt{6}}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}$ $y=2x$

$x=\frac{2\sqrt{18}}{3}$ <9, 2 $y=2(2\sqrt{2})$

$x=\frac{6\sqrt{2}}{3}$ $y=4\sqrt{2}$

$x=2\sqrt{2}$