CHAPTER 9.5 | THE DISTANCE FORMULA!

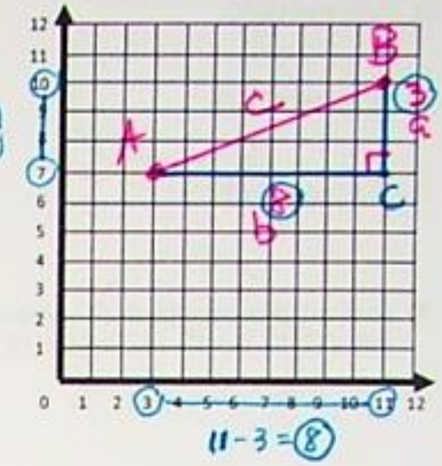
GETTING STARTED!

PLOT: A (3, 7) and B (11, 10)

Draw $\overline{AB}$.

Draw a horizontal segment from A to the right, and draw a vertical segment downward from B.

$10-7 = 3$



1) What is the coordinate of the point where the two segments intersect? (11, 7)

2) What is the length of AC? $\boxed{8u}$ BC? $\boxed{3u}$

3) How would you find the length of AB? $c = \sqrt{a^2 + b^2}$

The Pythagorean Thm!

4) Find the length of AB.

$a = 3 \quad b = 8 \quad c = ?$

$c = \sqrt{3^2 + 8^2} = \sqrt{9 + 64} = \boxed{\sqrt{73}}$

COOPERATIVE LEARNING! Work with your table partner and complete the following problems.

Record Theorem 71 below. (Page 393)

A(3,7) B(11,10) Using the Distance Formula with same points!

Theorem 71: $D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $c = \sqrt{a^2 + b^2}$	$D = \sqrt{(11-3)^2 + (10-7)^2}$ $= \sqrt{(8)^2 + (3)^2}$ $= \sqrt{64 + 9} = \sqrt{73}$ ✓

WRITING TO LEARN! Question: What do the following equations mean in your own words?

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	$d = \sqrt{(\Delta x)^2 + (\Delta y)^2}$
The distance between two points is the square root of the sum of the square of the horizontal & vertical differences	The distance between two points is the sq. rt of the sum of the change in x squared plus the change in y squared

For each pair of points, state whether Theorem 71 is needed to find the distance.

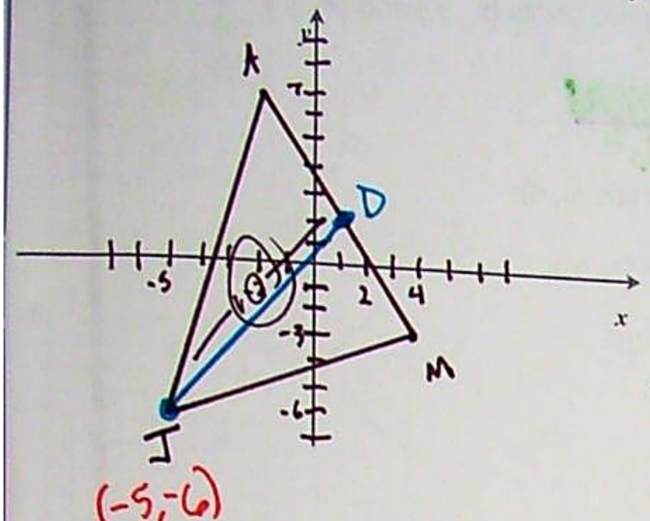
	(Circle)		Give an explanation for each answer.
1) (2, -4); (2, -9)	YES	NO	x-values same → vert line $\lvert -9-(-4)\rvert = \lvert -9+4\rvert = \lvert -5\rvert = 5u$
2) (-3, -5); (8, -5)	YES	NO	y-values same → horiz. line: $\lvert -3-8\rvert = \lvert -11\rvert = 11u$
3) (-1, -5); (3, 4)	YES	NO	$D = \sqrt{(-1-3)^2 + (-5-4)^2} = \sqrt{(-4)^2 + (-9)^2} = \sqrt{16+81} = \sqrt{97}$
4) (a, b); (6, 6)	YES	NO	$D = \sqrt{(a-6)^2 + (b-6)^2} = \sqrt{a^2 - 12a + b^2 - 12b + 72}$

Write a paragraph: *Explain how the distance formula is derived from the Pythagorean Theorem.*

On a coordinate plane, the horizontal and vertical distances between two points is the slope (rise over run → or just run and rise measures). Those distances create the LEGS of a right triangle and then the third side is the HYPOTENUSE. The hypotenuse represents the distance between the two points. Therefore, the distance formula is a mathematical way to apply the Pythagorean Theorem on a coordinate plane!

CLASSiC

2. For ΔJAM, with A (-2,7), J (-5, -6), and M (4, -3), find the length of the median from J. (This problem is so sick you might say it's ridonkulous....)



mdpt of $\overline{AM}$

$\left(\frac{-2+4}{2}, \frac{7+(-3)}{2}\right)$

$\left(\frac{2}{2}, \frac{4}{2}\right)$

D (1, 2)

$$D = \sqrt{(x-x)^2+(y-y)^2}$$
$$= \sqrt{(-5-1)^2+(-6-2)^2}$$
$$= \sqrt{(-6)^2+(-8)^2}$$
$$= \sqrt{36+64}$$
$$= \sqrt{100}$$
$$\boxed{10u}$$

3. A circle has its center at $\left(6\sqrt{3}, 4\sqrt{5}\right)$. A point on the circle is $\left(10\sqrt{3}, 6\sqrt{5}\right)$. Find the area and circumference of this circle.

Another Gem!

(*Remember:* $A_{\circ} = \pi r^2$ & $C_{\circ} = 2\pi r$)

$\pi(\sqrt{68})^2$ $\quad$ $2(2\sqrt{17})\pi$

$\boxed{A = 68\pi u^2}$ $\quad$ $\boxed{C = 4\sqrt{17}\pi u}$

$$(\text{radius})\, D = \sqrt{(10\sqrt{3}-6\sqrt{3})^2+(6\sqrt{5}-4\sqrt{5})^2}$$
$$= \sqrt{\underbrace{(4\sqrt{3})^2}_{4^2 \cdot 3}+\underbrace{(2\sqrt{5})^2}_{4 \cdot 5}}$$
$$= \sqrt{48+20}$$
$$= \sqrt{68} \quad \begin{matrix} 4 \\ 17 \end{matrix}$$
$$r = 2\sqrt{17}$$