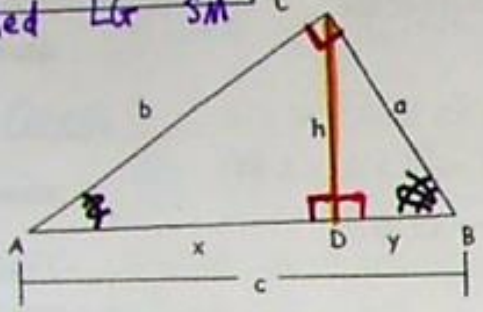
9.3 NOTES: Altitude-on-Hypotenuse Theorems

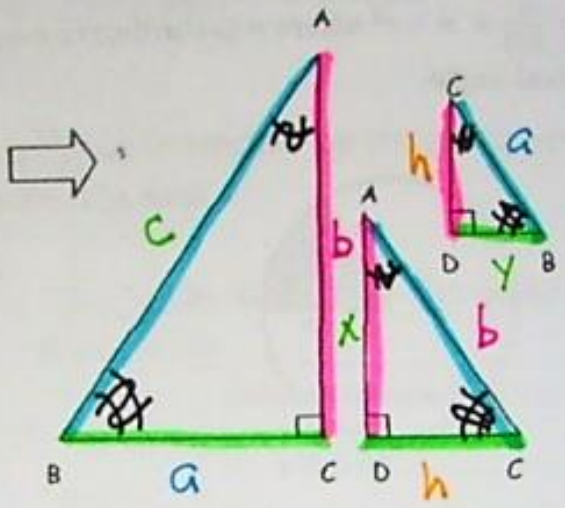
THM: If an altitude is drawn to the hypotenuse of a right triangle, then...

a. The two triangles formed are similar to the given right triangle and to each other.

$\Delta ADC \sim \Delta ACB \sim \Delta CDB$

Med Lg Sm





SAS~

AA~

SSS~

b. The altitude to the hypotenuse is the mean proportional between the segments of the hypotenuse

$\frac{x}{h} = \frac{h}{y}$, or $h^2 = xy$

heartbeat!

c. Either leg of the given right triangle is the mean proportional between the hypotenuse of the given right triangle and the segment of the hypotenuse adjacent to that leg (i.e., the projection of that leg on the hyp.)

$\frac{y}{a} = \frac{a}{c}$, or $a^2 = yc$; and $\frac{x}{b} = \frac{b}{c}$, or $b^2 = xc$

SUMMARY:

$h^2 = x \cdot y$

$b^2 = x \cdot c$

$a^2 = y \cdot c$

$\frac{\text{Leg}}{\text{hyp}} = \frac{\text{Leg}}{\text{hyp}}$

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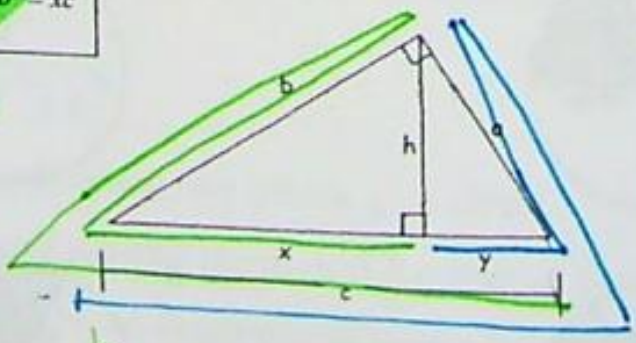


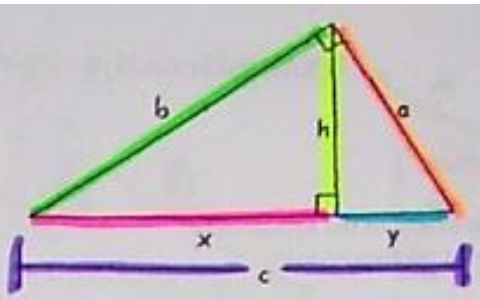
boomerang!

SUMMARY:

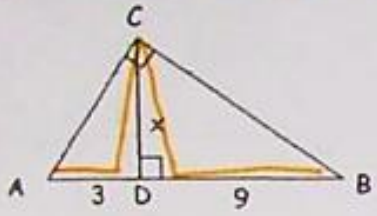
$h^2 =$ [illegible]

$b^2 =$ [illegible]

$a^2 =$ [illegible]



EX1. If AD = 3 and DB = 9, find CD.



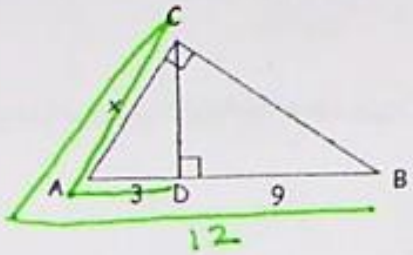
~~$\frac{3}{x} = \frac{x}{9}$~~ ✓ $\frac{3}{\sqrt{27}} = \frac{\sqrt{27}}{9}$

$\sqrt{x^2} = \sqrt{3 \cdot 9}$

$x = 3\sqrt{3}$

heartbeat!

EX2. If AD = 3 and DB = 9, find AC.



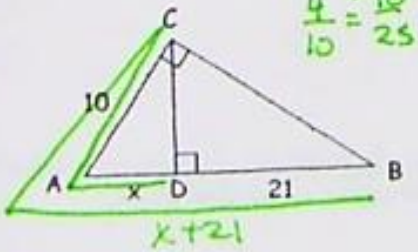
~~$\frac{3}{x} = \frac{x}{12}$~~ ✓ $\frac{3}{6} = \frac{6}{12}$

$\sqrt{x^2} = \sqrt{36}$

$x = 6$

boomerang!

EX3. If DB = 21 and AC = 10, find AD.



~~$\frac{4}{10} = \frac{10}{25}$~~ ✓

$\frac{x}{10} = \frac{10}{x+21}$

$x^2 + 21x = 100$

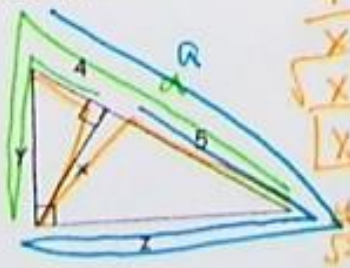
$x^2 + 21x - 100 = 0$

$(x+25)(x-4) = 0$

$x = \{-25, 4\}$

boomerang!

EX4. Find x, y and z.



$\frac{4}{x} = \frac{x}{5}$

$\sqrt{x^2} = \sqrt{4 \cdot 5}$

$x = 2\sqrt{5}$

$\frac{4}{\sqrt{20}} = \frac{\sqrt{20}}{5}$ ✓

$\frac{4}{y} = \frac{y}{9}$

$\sqrt{y^2} = \sqrt{36}$

$y = 6$

$\frac{4}{6} = \frac{6}{9}$ ✓

$\frac{5}{z} = \frac{z}{9}$

$\sqrt{z^2} = \sqrt{5 \cdot 9}$

$z = 3\sqrt{5}$

$\frac{5}{\sqrt{45}} = \frac{\sqrt{45}}{9}$ ✓