8-5 Problem Solving With Trigonometry

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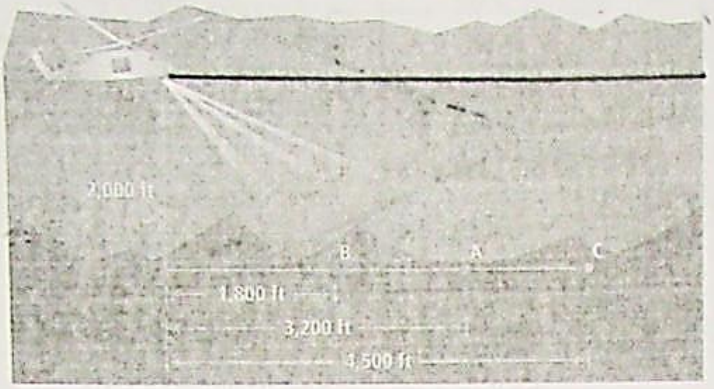
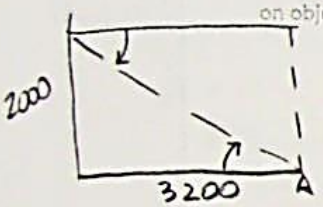
MODEL & DISCUSS

A search-and-rescue team is having a nighttime practice drill. Two members of the team are in a helicopter that is hovering at 2,000 feet above ground level.



A. The team first tries to locate object A. At what angle from the horizontal line even with the helicopter should they position the spotlight so that it shines on object A?

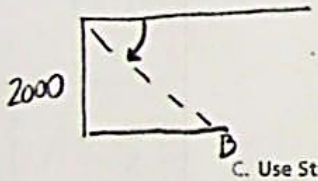


$\tan A = \frac{2000}{3200}$

$A = \tan^{-1}(5 \div 8)$

$m\angle A \approx 32°$

B. Next, they shine the spotlight on object B. How does the angle of the spotlight from the horizontal line change?



The angle of depression increases.

C. **Use Structure** In general, how does the angle of the spotlight from the horizontal change as the light moves from object A to object B? From object A to object C? MP.7

The angle of depression increases for objects that are closer to the helicopter and decreases to spot objects farther away.

HABITS OF MIND

Use Structure What geometric figures are useful in modeling situations where you want to find angle measures? Why are these figures helpful? MP.7

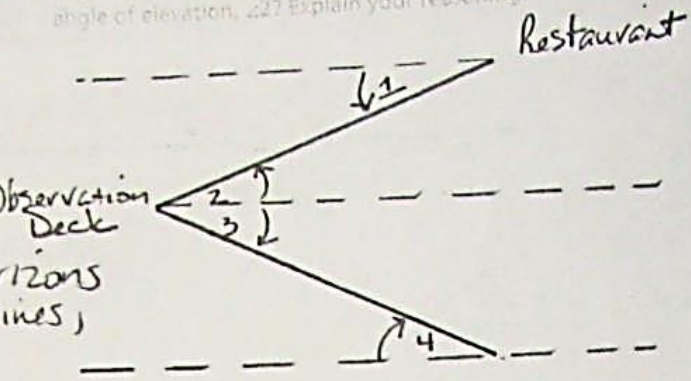
Triangles! missing side lengths and angle measures can be determined by using the Pythagorean Thm, Trigonometric ratios, the Law of Sines, and the Law of Cosines.

EXAMPLE 1 **Try It!** Identify Angles of Elevation and Depression

1. In Example 1, how does the angle of depression, ∠1, compare with the angle of elevation, ∠2? Explain your reasoning.

(AIA ≅)

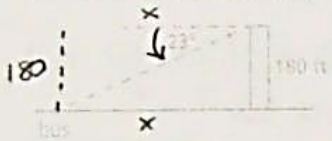
∠1 and ∠2 are ≅ since the horizons are parallel lines, they are ≅ alternate interior angles



EXAMPLE 2 **Try It!** Use Angles of Elevation and Depression

2. Nadeem sees the tour bus from the top of the tower. To the nearest foot, how far is the bus from the base of the tower?

424 ft



$$\tan 23 = \frac{180}{x}$$

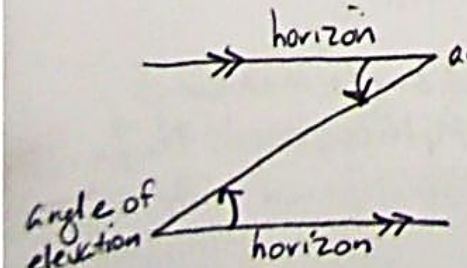
$$x \tan 23 = 180$$

$$x = \frac{180}{\tan 23}$$

$$x \approx 424 \text{ ft}$$

HABITS OF MIND

Communicate Precisely How can you relate angles of elevation and depression to parallel lines intersected by a transversal? MP.6



The horizons are parallel to each other, and the "line of sight" whether an angle of elevation or depression, is the transversal.

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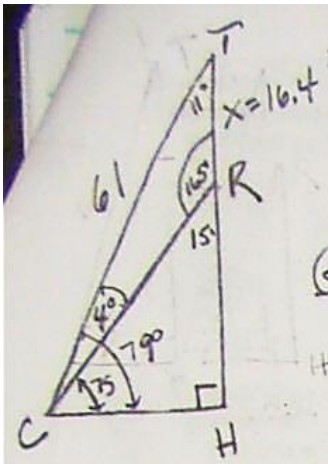


EXAMPLE 3 **Try It! Use Trigonometry to Solve Problems**

45.0

3. In Example 3, how far is the student from the instructor at the resting point?

$\frac{\sin 4}{x} = \frac{\sin 165}{61}$

$\frac{(\sin 165)x}{\sin 165} = \frac{61(\sin 4)}{\sin 165}$

$x \approx 16.4$ ft

$\angle T = 180 - (165+4) = 11°$

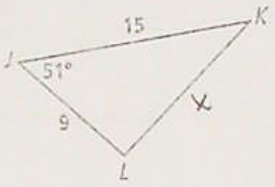
$\frac{\sin 165}{61} = \frac{\sin 11}{CR}$

$\frac{CR(\sin 165)}{\sin 165} = \frac{61(\sin 11)}{\sin 165}$

$CR \approx 45$ ft

EXAMPLE 4 **Try It! Use Trigonometry to Find Triangle Area**

4. a. What is the area of $\triangle JKL$?



$A = \frac{1}{2}(9)(15)(\sin 51)$

$= \frac{1}{2}(135)(\sin 51)$

$\approx 52.5u^2$

$52.46u^2$

Or use Law of Cosines, then Heron's Formula.

$x^2 = 9^2 + 15^2 - 2(9)(15)(\cos 51)$

$81 + 225 - 270(\cos 51)$

$306 - 169.917$

$\sqrt{136.083}$

11.7

$P = 9+15+11.7$

$24+11.7$

35.7

$s = 17.85$

$A = \sqrt{17.85(17.85-15)(17.85-9)(17.85-11.7)}$

$A = \sqrt{17.85(2.85)(8.85)(6.15)}$

$A = \sqrt{2768.863}$

$A = 52.6u^2$

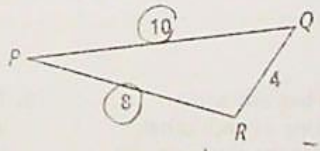
$15.20u^2$

$A = \frac{1}{2}bc(\sin A)$

$A = \frac{1}{2}ac(\sin B)$

$A = \frac{1}{2}ab(\sin C)$

b. What is the area of $\triangle PQR$? *Hint:* First apply the Law of Cosines to find the measure of the angle included between $\overline{PQ}$ and $\overline{PR}$. Then apply the area formula with the sine of the angle measure.



$4^2 = 8^2 + 10^2 - 2(8)(10)\cos P$

$16 = 164 - 160(\cos P)$

$\frac{-148}{-160} = \frac{-160(\cos P)}{-160}$

$.925 = \cos P$

$P = \cos^{-1}(.925)$

$P = 22.3°$

$A = \frac{1}{2}(10)(8)(\sin 22.3)$

$= 40(\sin 22.3)$

$A \approx 15.2u^2$

HABITS OF MIND

Reason What quantities do you need to know if you want to apply the Law of Sines or Cosines to solve a problem? MP.2

Law of Sines: Need any two angles and a side or 2 sides with an opposite angle

Law of Cosines: SAS or SSS