8.5

Thm 6.5

Triangle Proportionality Theorem (SIDE-SPLITTER THM) If a line parallel to one side of a triangle intersects the other two sides, then it divides those sides proportionally.

Find the value of x.

1. $\frac{33}{x} = \frac{27}{18}$; $\frac{3x}{3} = \frac{66}{3}$; $x = 22$

2. $\frac{12}{9} = \frac{4}{3} = \frac{16}{x}$; $\frac{4x}{4} = \frac{48}{4}$; $x = 12$

3. $\frac{10}{6} = \frac{5}{3} = \frac{7.5}{x}$; $\frac{5x}{5} = \frac{22.5}{5}$; $x = 4.5$

4. $\frac{12}{15} = \frac{4}{5} = \frac{x}{20}$; $\frac{5x}{5} = \frac{80}{5}$; $x = 16$

5. $\frac{10}{4} = \frac{5}{2} = \frac{x}{6}$; $\frac{2x}{2} = \frac{30}{2}$; $x = 15$

6. $\frac{4}{12} = \frac{1}{3} = \frac{20-x}{x}$; $60 - 3x = x$; $\frac{-4x}{-4} = \frac{-60}{-4}$; $x = 15$

Thm 6.6

If three parallel lines intersect two transversals, then they divide the transversals proportionally.

Find the value of x.

7. $\frac{x}{12.5} = \frac{12}{15} = \frac{4}{5}$; $\frac{5x}{5} = \frac{50}{5}$; $x = 10$

8. $\frac{5}{4} = \frac{x-5}{8}$; $4x - 20 = 40$; $\frac{4x}{4} = \frac{60}{4}$; $x = 15$

9. $\frac{27}{18} = \frac{3}{2} = \frac{x+10}{x}$; $3x = 2x + 20$; $x = 20$

Thm 6.7

Triangle Angle-Bisector Theorem If a ray bisects an angle of a triangle, then it divides the opposite side into segments proportional to the other two sides.

Find the value of x.

10. $\frac{\text{seg}}{\text{side}}$ $\frac{x}{24} = \frac{12}{18} = \frac{2}{3}$; $\frac{3x}{3} = \frac{48}{3}$; $x = 16$

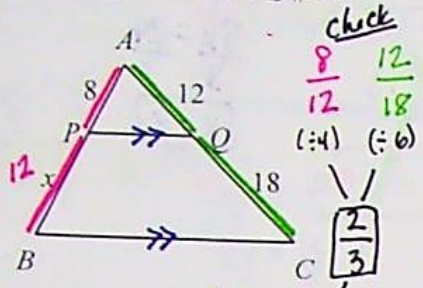
11. $\frac{\text{side}}{\text{seg}}$ $\frac{x}{8} = \frac{12.5}{10}$; $\frac{10x}{10} = \frac{100}{10}$; $x = 10$

12. $\frac{\text{side}}{\text{seg}}$ $\frac{10}{x} = \frac{20}{15-x}$; $20x = 150 - 10x$; $\frac{30x}{30} = \frac{150}{30}$; $x = 5$

Theorem Worksheet (8.5)

Theorem 65: (Triangle Proportionality Theorem)
If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally. (a.k.a.: Side-Splitter Theorem)

① Find x given that $\overline{PQ} \parallel \overline{BC}$.



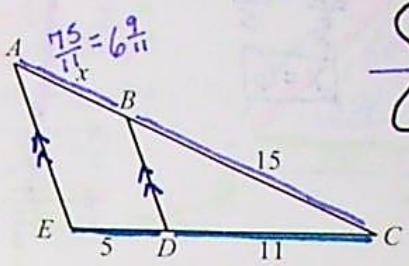
Check: $\frac{8}{12}$ $\frac{12}{18}$ (÷4) (÷6) → $\frac{2}{3}$

$\frac{8}{x} = \frac{12}{18}$ (2, 3)

$\frac{8x}{8} = \frac{24}{2}$

$x = 12$

② Find x given that $\overline{AE} \parallel \overline{BD}$.



Check: 75 ✓ 75; $\frac{75}{11}$ 5; 15 11

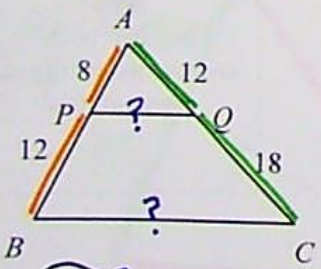
$\frac{x}{15} = \frac{5}{11}$

$11x = 75$

$x = 6\frac{9}{11}$

Converse of Theorem 65: (Side-Splitter Converse)
If a line divides two sides of a triangle proportionally, then it is parallel to the third side.

③ Is PQ parallel to BC?

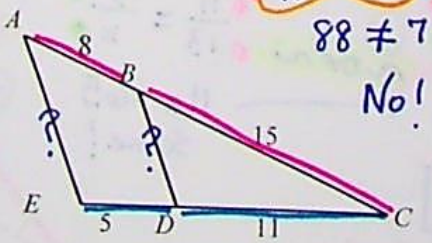


$\frac{8}{12} \stackrel{?}{=} \frac{12}{18}$

$144 = 144$

Yes...by Side-Splitter Thm
$\overline{PQ} \parallel \overline{BC}$!

④ Is AE parallel to BD?



$\frac{8}{15} \stackrel{?}{=} \frac{5}{11}$

$88 \neq 75$

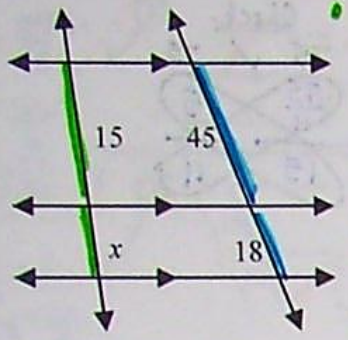
No! $\overline{AE} \nparallel \overline{BD}$

How is the "Midline Theorem" (7.1) a bit different from the "Side-Splitter Theorem" (8.5)?

The endpoints of a Midline are midpoints of two sides of Δ. Both split sides of Δ into segments that are proportional, and the segment is ∥ to third side. Whether a Midline or Sidesplitter Seg.

Theorem 66: (Proportionality with Parallel Lines and Tranversal Lines)
If three parallel lines intersect two transversals, then they divide the transversals proportionally. [Related to Side-Splitter Theorem - see page 351)

(5) Solve for x.

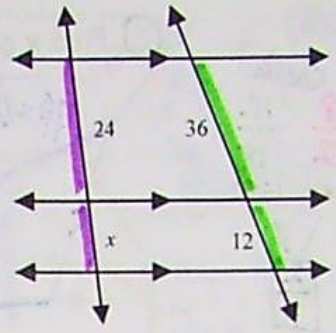


$\frac{15}{x} = \frac{45}{18}$ $\frac{5}{2}$

$\frac{5x}{5} = \frac{30}{5}$

$x = 6$

(6) Solve for x.



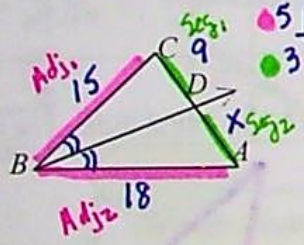
$\frac{24}{x} = \frac{36}{12}$ $\frac{3}{1}$

$\frac{8x}{8} = \frac{24}{3}$

$x = 8$

Theorem 67: Triangle Angle Bisector Theorem
If a ray bisects an angle of a triangle, then it divides the opposite side into segments whose lengths are proportional to the lengths of the other two sides of the angle.

(7) Find AD if $\overline{BD}$ bisects $\angle ABC$, BC = 15, CD = 9, and AB = 18.



$\frac{15}{9} = \frac{18}{x}$ $\frac{5}{3}$

$\frac{5x}{5} = \frac{54}{5}$

$x = 10.8$

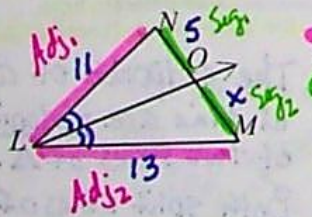
~ or ~

$\frac{15}{18} = \frac{9}{x}$ $\frac{5}{6}$

$5x = 54$

$x = 10.8$

(8) Find *OM* if $\overline{LO}$ bisects $\angle NLM$, $LM = 13$, $NO = 5$, and $LN = 11$.



$\frac{11}{5} = \frac{13}{x}$

$\frac{11x}{11} = \frac{65}{11}$

$x = 5\frac{10}{11}$

~ or ~

$\frac{11}{13} = \frac{5}{x}$

$11x = 65$ same!

Proportions:
Set-up
~ Either ~

$\frac{\text{Adj 1}}{\text{Adj 2}} = \frac{\text{Seg 1}}{\text{Seg 2}}$

~ or ~

$\frac{\text{Adj 1}}{\text{Seg 1}} = \frac{\text{Adj 2}}{\text{Seg 2}}$

AND
They can both be overturned or flipped around, too!

Geometry: 8.5 Proportion Practice Name _____

Find all variables for each problem.

* Be Careful!

By Similar Δ's $\frac{6}{14}=\frac{3}{7}$

1)

Side-Splitter

$\frac{2}{3}=\frac{y}{6}$

$\frac{3y}{3}=\frac{12}{3}$

$y=4$

Side-Splitter

SF: $\frac{6}{8}=\frac{3}{4}$

$\frac{3}{4}=\frac{y}{21-y}$

$4y=63-3y$

$\frac{7y}{7}=\frac{63}{7}$

$y=9$

(*) ~Δ's

$\frac{2}{x}=\frac{5}{7.5}$

$\frac{5x}{5}=\frac{15}{5}$

$x=3$

2)

$\frac{3}{7}=\frac{y}{21}$

$\frac{7y}{7}=\frac{63}{7}$

$y=9$

$\frac{3}{7}=\frac{x}{16}$ (*)

$\frac{7x}{7}=\frac{48}{7}$

$x=6\frac{6}{7}$

3)

SF: $\frac{x}{36}=\frac{3}{4}$

$\frac{3}{4}=\frac{35}{x}$

$\frac{3x}{3}=\frac{140}{3}$

$x=46\frac{2}{3}$

SF: $\frac{y}{20}=\frac{2}{5}$

$\frac{5y}{5}=\frac{40}{5}$

$y=8$

4)

$\frac{5}{4}=\frac{20}{2y-1}$

$10y-5=80$

$\frac{10y}{10}=\frac{85}{10}$

$y=8.5$

$\frac{3}{x+4}=\frac{x}{15}$

$x^2+4x=45$

$x^2+4x-45=0$

$(x+9)(x-5)=0$

$x=\{-9, 5\}$

$x=5$

oops! #5 is missing

6)

$\frac{x}{8}=\frac{12.5}{10}$

$\frac{10x}{10}=\frac{100}{10}$

$x=10$

7)

$\frac{10}{x}=\frac{20}{15-x}$

$20x=150-10x$

$\frac{30x}{30}=\frac{150}{30}$

$x=5$