Activity

8-4 The Law of Cosines

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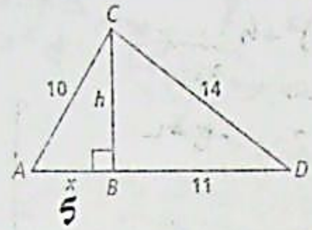
EXPLORE & REASON

Use $\triangle ABC$ to answer the questions.



A. Write equations for the side lengths of $\triangle ABC$ and $\triangle CBD$ using the Pythagorean Theorem.

$\triangle ABC$

$x^2+h^2=10^2$

$h^2=10^2-x^2$

$h^2=10^2-x^2$

$\triangle CBD$

$h^2+11^2=14^2$

$h^2=14^2-11^2$

$h^2=14^2-11^2$

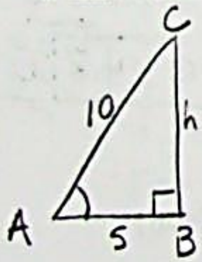
B. Use a system of equations to solve for x.

$$10^2-x^2=14^2-11^2$$
$$-x^2=14^2-11^2-10^2$$
$$-x^2=196-121-100$$
$$\frac{-x^2}{-1}=\frac{-25}{-1}$$

$\sqrt{x^2}=\sqrt{25}$

$\boxed{x=5}$

C. Use Structure How can you use the information you found to determine $m\angle A$? MP.7



Yes, $\triangle ABC$ is a 30-60-90 Rt$\triangle$ since one leg is half the hypotenuse. $\measuredangle A$ must be $60°$ because the short leg (5) is across from $\measuredangle C$ ($30°$).

Or use $\cos A = \frac{5}{10} \Rightarrow \cos A = 0.5$, and $A = \cos^{-1}(.5)$

$\measuredangle A = 60°$

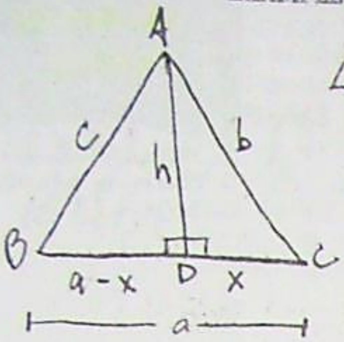
HABITS OF MIND

Look for Relationships Does constructing an altitude in a triangle always divide a triangle into similar triangles? Explain. MP.7

No – not necessarily. If an altitude is drawn to the hypotenuse of a right triangle, then the result is three similar right triangles. Or, if an altitude is dropped in an isosceles triangle, the resulting right triangles are similar/congruent to each other

EXAMPLE 1 **Try It!** Develop the Law of Cosines with Trigonometry

1. Use the same method as in Example 1 to write equations for a^2 using cos A and b^2 using cos B.



$\triangle ABD$

$c^2 = (a-x)^2 + h^2$

$c^2 = a^2 - 2ax + x^2 + h^2$

$c^2 = a^2 - 2ax + b^2$

Now: $\cos C = \frac{x}{b}$

and $x = b(\cos C)$

$c^2 = a^2 - 2a(b\cos C) + b^2$

rearrange: $\boxed{c^2 = a^2 + b^2 - 2ab(\cos C)}$

$\triangle ACD$

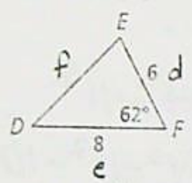
$b^2 = x^2 + h^2$

$a^2 = b^2 + c^2 - 2bc(\cos A)$

$b^2 = a^2 + c^2 - 2ac(\cos B)$

EXAMPLE 2 **Try It!** Use the Law of Cosines to Find a Side Length

2. a. What is DE?



$f^2 = 8^2 + 6^2 - 2(8)(6)(\cos 62)$

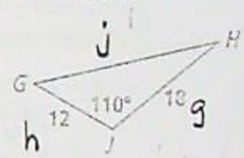
$f^2 = 64 + 36 - 96(\cos 62)$

$f^2 = 100 - 45.1$

$\sqrt{f^2} = \sqrt{54.9}$

$\boxed{f \approx 7.4}$

b. What is GH?



$j^2 = 12^2 + 18^2 - 2(12)(18)(\cos 110)$

$j^2 = 144 + 324 - 432(\cos 110)$

$j^2 = 468 - (-147.8)$

$\sqrt{j^2} = \sqrt{615.8}$

$\boxed{j \approx 24.8}$

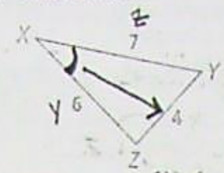
HABITS OF MIND

Construct Arguments Once you use the cosine to find the length of the third side, is it possible to find the measures of the other two angles? Explain. MP.3

Yes! The Law of Cosines can be used since all side lengths are known - or the Law of Sines can now be used since the only known angle is no longer between the only known sides.

EXAMPLE 3 **Try It!** Use the Law of Cosines to Find an Angle Measure

3. a. What is $m\angle X$?



$4^2 = 6^2 + 7^2 - 2(6)(7)(\cos x)$

$16 = 36 + 49 - 84(\cos x)$

$16 = 85 - 84(\cos x)$

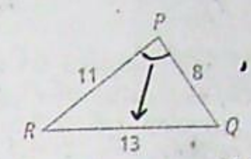
$-85 \quad -85$

$\frac{-69}{-84} = \frac{-84(\cos x)}{-84}$

$.8214 = \cos x$

$x = \cos^{-1}(.8214) \approx \boxed{34.8^\circ}$

b. What is $m\angle P$?



$13^2 = 8^2 + 11^2 - 2(8)(11)(\cos P)$

$169 = 64 + 121 - 176(\cos P)$

$169 = 185 - 176(\cos P)$

$-185 \quad -185$

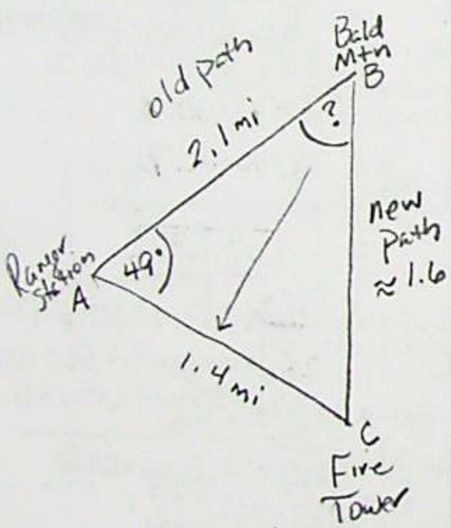
$\frac{-16}{-176} = \frac{-176(\cos P)}{-176}$

$.0909 = \cos P$

$P = \cos^{-1}(.0909) \boxed{\approx 84.8^\circ}$

EXAMPLE 4 **Try It!** Use the Law of Cosines to Solve a Problem

4. In Example 4, what is the angle that the new path forms with the old path at Bald Mountain?



$1.4^2 = 1.6^2 + 2.1^2 - 2(1.6)(2.1)(\cos B)$

$1.96 = 2.56 + 4.41 - 6.72(\cos B)$

$1.96 = 6.97 - 6.72(\cos B)$

$-6.97 \quad -6.97$

$\frac{-5.01}{-6.72} = \frac{-6.72(\cos B)}{-6.72}$

$.7455 = \cos B$

$B = \cos^{-1}(.7455) \boxed{\approx 41.8^\circ}$

HABITS OF MIND

Make Sense and Persevere How can you determine whether the Law of Cosines can be used to solve a real-world problem? MP.1

SSS - If the lengths of the three sides of a triangle are known... solve for ∡'s!

SAS - If two sides and the included angle are known, the Law of Cosines can be used to find the third side — then use Law of Sines for another angle.