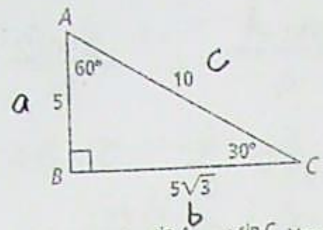
Activity

8-3
The Law of Sines

PearsonRealize.com

EXPLORE & REASON

Consider the 30°-60°-90° triangle shown.

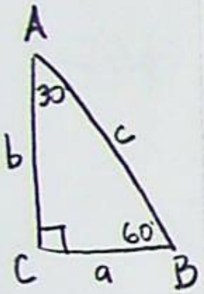


A. Calculate the values of the ratios $\frac{\sin A}{BC}$ and $\frac{\sin C}{AB}$. How are the values of the ratios related?

$$\frac{\sin A}{BC} = \frac{\frac{5\sqrt{3}}{10}}{\frac{5\sqrt{3}}{1}} = \frac{\cancel{5\sqrt{3}}}{10} \cdot \frac{1}{\cancel{5\sqrt{3}}} = \frac{1}{10} \quad (\sin 60)$$

$$\frac{\sin C}{AB} = \frac{\frac{5}{10}}{\frac{5}{1}} = \frac{5}{10} \cdot \frac{1}{5} = \frac{1}{10} \quad (\sin 30)$$

The ratios are equal



$\sin 60 = \frac{b}{c}$

$\sin 30 = \frac{a}{c}$

$\frac{\frac{b}{c}}{b} \quad \frac{\frac{a}{c}}{a}$

$\frac{\cancel{b}}{c} \cdot \frac{1}{\cancel{b}} \quad \frac{\cancel{a}}{c} \cdot \frac{1}{\cancel{a}}$

$\frac{1}{c} = \frac{1}{c}$

B. Make Sense and Persevere Do you think the ratios would have the same relationship in any 30°-60°-90° right triangle? Explain your answer. MP.1

Yes - since the side ratios for any 30-60-90 triangle follow the ratio $1 - \sqrt{3} - 2$, the law of sines states that the sine ratio for any angle when divided by the side length opposite the reference angle will always be the same for any angle of the triangle.

HABITS OF MIND

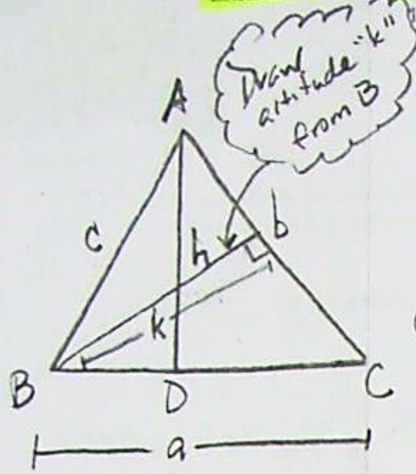
Look for Relationships What general patterns or relationships seem to occur in a triangle between each angle and its opposite side? MP.7

The comparison (ratio) of the sine of the angle to the length of the opposite side from the angle is the same. for every angle of the triangle.

Notes

EXAMPLE 1 **Try It! Explore the Sine Ratio**

1. For Example 1, show that $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.



$$\frac{\sin A}{a} \qquad \frac{\sin B}{b} \qquad \frac{\sin C}{c}$$

$$\sin A = \frac{k}{c} \qquad \sin B = \frac{h}{c} \qquad \sin C = \frac{k}{a}$$

$$c(\sin A) = k \qquad a(\sin C) = k$$

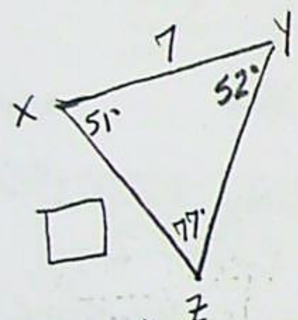
$$\frac{\cancel{c}(\sin A)}{\cancel{c}\,a} = \frac{\cancel{a}(\sin C)}{\cancel{a}\,c}$$

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

If $\frac{\sin B}{b} = \frac{\sin C}{c}$ and $\frac{\sin C}{c} = \frac{\sin A}{a}$, then by transitive property $\frac{\sin A}{a} = \frac{\sin B}{b}$

EXAMPLE 2 **Try It! Use the Law of Sines to find a Side Length**

2. In Example 2, what is XZ to the nearest tenth?



$180 - (51 + 77)$
$180 - 128$
$52°$

$$\frac{\sin 77}{7} = \frac{\sin 52}{XZ}$$

$$\frac{.9744}{7} = \frac{.788}{XZ}$$

$$\frac{\cancel{.9744}(XZ)}{\cancel{.9744}} = \frac{5.516}{.9744}$$

$$\boxed{\overline{XZ} \approx 5.7u}$$

HABITS OF MIND

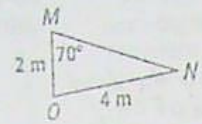
Reason How can you use the Law of Sines if given the measures of two angles and the length of the included side? MP.2

As in "Try It" #2, if you know two angle measures, subtract thier sum from 180 to find the third angle.
[Triangle Sum Theorem]

Notes

EXAMPLE 3 Try It! Use Law of Sines to Find the Measure of an Angle

3.



a. What is $m\angle N$?

$$\frac{\sin N}{2} = \frac{\sin 70}{4}$$

$$\frac{\sin N}{2} = \frac{.9397}{4}$$

$$\sin N = 2(.2349)$$

$$m\angle N \approx \sin^{-1}(.4698)$$

$$\boxed{m\angle N \approx 28^\circ}$$

b. What is $m\angle O$?

$$180-(70+28)$$

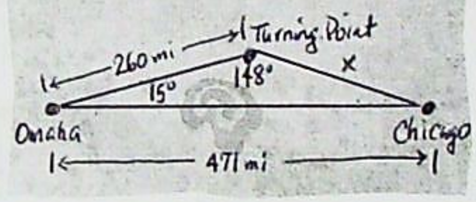
$$180-(98)$$

$$\boxed{82^\circ}$$

EXAMPLE 4 Try It! Apply the Law of Sines

4. Suppose the pilot chose to fly north of the storm. How much farther is that route than the direct route?

around storm $(260+x)$ miles



$$\begin{array}{r} 260 \\ +\ 230.1 \\ \hline 490.1 \end{array}$$

$$\frac{\sin 15}{x} = \frac{\sin 148}{471}$$

$$\frac{.2588}{x} = \frac{.5299}{471}$$

$$471(.2588) = .5299x$$

$$\frac{121.9}{.5299} = \frac{\cancel{.5299}x}{\cancel{.5299}}$$

$$230.1 \approx x$$

$$\begin{array}{rl} 490.1 & \text{(around)} \\ -\ 471.0 & \text{(direct)} \\ \hline \end{array}$$

$\boxed{\text{19.1 miles longer}}$

HABITS OF MIND

Communicate Precisely What information do you need in order to use the Law of Sines to solve a problem? MP.6

You must know:

either • two angles and length of one side

or • two sides and measure of one <u>non-included</u> angle.