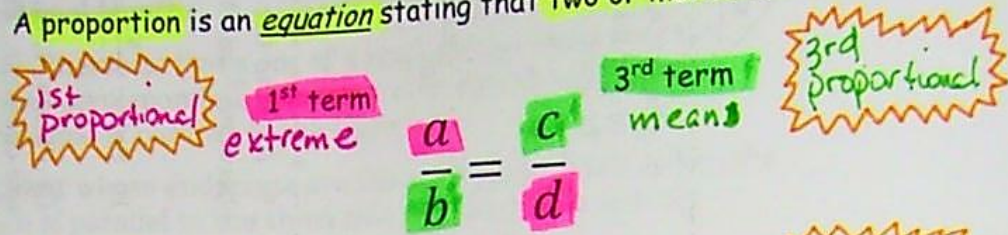**Definition:** A ratio is a quotient of two numbers
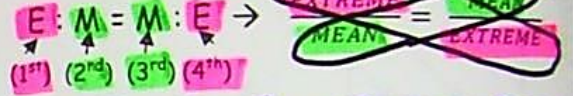
Example: $\frac{3}{2}$ or $\frac{2}{3}$ ... $\frac{3}{2} > \frac{2}{3}$, right?   (the *different* values mean the ratios are making *different comparisons*)

1.5 $0.\overline{6}$

Definition: A proportion is an *equation* stating that two or more ratios are EQUAL.

1st proportional — 1st term — extreme

3rd term — means — 3rd proportional

$$\frac{a}{b} = \frac{c}{d}$$

2nd proportional — 2nd term — mean

4th term — extreme — 4th proportional

Factoid: 1st and 4th terms are called the EXTREMES   E : M = M : E → $\frac{\text{EXTREME}}{\text{MEAN}} = \frac{\text{MEAN}}{\text{EXTREME}}$
Factoid: 2nd and 3rd terms are called the MEANS   (1st) (2nd) (3rd) (4th)

Th^m : In a proportion, the product of the means is equal to the product of the extremes.
THEOREM   (Means-Extremes Products Theorem)

Example: If $a{:}b = c{:}d$, then $\frac{a}{b} = \frac{c}{d}$, and $ad = bc$

Theorem

Th^m : If the product of a pair of nonzero numbers is equal to the product of another pair of nonzero numbers, then either pair of numbers may be made the extremes and the other pair the means of a proportion. [EM = ME]

$\frac{4}{6} = \frac{6}{9}$ $\frac{6}{4} = \frac{9}{6}$ (3)(4) $\frac{3}{2} = \frac{6}{4}$ | $\frac{2}{3} = \frac{4}{6}$

If Geometric mean (is 6)

Example: If $pq = rs$, then $\frac{p}{r} = \frac{s}{q}$, $\frac{p}{s} = \frac{r}{q}$, $\frac{r}{p} = \frac{q}{s}$! All result in $p \cdot q = r \cdot s$

Numerically: If $pq = rs$ ....

And,

$p = 3, q = 4, r = 2$ and $s = 6$.

Then: $(3)(4) = (2)(6)$ and $\frac{3}{2} = \frac{6}{4}$ and $\frac{3}{6} = \frac{2}{4}$ and $\frac{2}{3} = \frac{4}{6}$!!!

$1.5 = 1.5$ $0.5 = 0.5$ $0.\overline{6} = 0.\overline{6}$

1

8.1 Ratio and Proportion

Geometry Notes and Class Examples

ARITHMETIC MEAN

Definition: An ARITHMETIC MEAN is the _average_ of two or more numbers.

Example: Find the arithmetic mean of 80 and 90, and then for -25 and 75

$$\bar{X} = \frac{80+90}{2} = \frac{170}{2} = 85 \qquad \bar{X} = \frac{-25+75}{2} = \frac{50}{2} = 25$$

Question: Where is the mean value of a set of data ALWAYS located?

Answer: _Between the two extremes_

GEOMETRIC MEAN

In a **MEAN PROPORTION**, the means are the same.

$$\frac{4}{8} = \frac{8}{16}$$
$$\frac{4}{x} = \frac{x}{16}$$

Example: If $1 : 4 = 4 : 16,$ ... then $\frac{1}{4} = \frac{4}{16}$ ... so $\frac{a}{x} = \frac{x}{r}$ ... and $x^2 = ar$

(4)(16)

$\sqrt{x^2} = \sqrt{64}$

In words: One is to Four as Four is to Sixteen

$\frac{1}{4} = \frac{16}{64}$ NOT a mean proportion

How do you solve this? for x

Definition: If the means in a proportion are equal, either the mean is called a _geometric_ mean or mean _Proportioned_ between the extremes.

$$\frac{3}{2} = \frac{6}{4}$$

Properties of Proportions

Question: The proportion $\frac{a}{b} = \frac{c}{d}$ is equivalent to?   Answer: _All equivalent_

✓ A) $ad = bc$ ✓ B) $\frac{b}{a} = \frac{d}{c}$ ✓ C) $\frac{a}{c} = \frac{b}{d}$ ✓ D) $\frac{a+b}{b} = \frac{c+d}{d}$

$$\frac{3+2}{2} = \frac{6+4}{4}$$
$$\frac{5}{2} = \frac{10}{4}$$

Example 1: If $\frac{x}{y} = \frac{5}{6}$, complete each statement:

a. $6x = $ _5y_

b. $\frac{y}{x} = \frac{6}{5}$ "Overturning"

c. $\frac{x}{5} = \frac{y}{6}$ "Cross-ways"

d. $\frac{x+y}{y} = \frac{5+6}{6}$ "crazy"

Example 2: Solving for a variable.
Solve for x:

a. $\frac{x}{5} = \frac{12}{7}$

$\frac{7x}{7} = \frac{60}{7}$

$x = \frac{60}{7}$

check $\frac{60}{7} = \frac{12}{7}$

$\frac{60}{7} \cdot \frac{12}{5} = \frac{12}{7}$

60 ✓ 60

b. $\frac{x+3}{8} = \frac{x}{4}$

4(x+3)

check $\frac{6}{8} = \frac{3}{4}$

24 = 24 ✓ 2

c. $\frac{18}{x+6} = \frac{6}{x}$

$8x = 4x + 12$
$-4x \quad -4x$
$\frac{4x}{4} = \frac{12}{4}$
$x = 3$

check $\frac{18}{9} = \frac{6}{3}$

6(x+6) $18x = 6x + 36$
$-6x \quad -6x$
$\frac{12x}{12} = \frac{36}{12}$
$x = 3$

$54 = 54$ ✓

8.1 Ratio and Proportion $\frac{2}{4} = \frac{4}{8}$ $\frac{2}{x} = \frac{x}{8}$ Geometry Notes and Class Examples

$x^2 = 16$ $\sqrt{x^2} = \pm\sqrt{16}$

$x = \pm 4$

IMPORTANT Note!

There are __TWO__ possible values of the geometric mean. If we are calculating the geometric mean to find a missing segment length in a GEOMETRY problem, we must reject the __negative__ geometric mean because __negative measures do not make sense__

If we are calculating the geometric mean in an ALGEBRA problem, we state __both (±)__

Example 3: Find the geometric mean between each pair of numbers.

ⓐ 3 and 10

$$\frac{3}{x} = \frac{x}{10}$$
$\frac{3}{-\sqrt{30}} = \frac{-\sqrt{30}}{10}$ $\sqrt{x^2} = \sqrt{30}$
$x = \pm\sqrt{30}$

(b.) 10 and 20

$$\frac{10}{x} = \frac{x}{20}$$
$\frac{10}{-\sqrt{200}} = \frac{-\sqrt{200}}{20}$ $\sqrt{x^2} = \sqrt{200}$
$x = \pm\sqrt{200} = \pm 10\sqrt{2}$

(c) 10 and 40

$$\frac{10^-}{-20x} = \frac{x \cdot 20}{40}$$
$\sqrt{x^2} = \sqrt{400}$
$x = \pm 20$

Example 4: Find the positive arithmetic and geometric means between each pair of numbers. State which mean is greater in each case (hw #14)

a. 20 and 80

Ⓐ $\frac{20+80}{2}$
$\frac{100}{2}$
(50)

Ⓖ $\frac{20}{x} = \frac{x}{80}$
$\sqrt{x^2} = \sqrt{1600}$
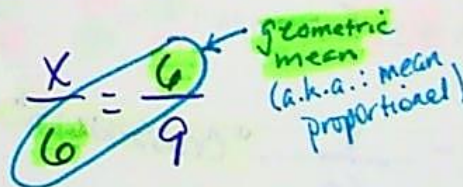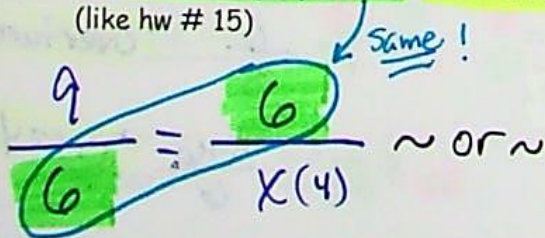(x = 40)

b. 4 and 9

Ⓐ $\frac{4+9}{2}$
$\frac{13}{2}$
(6.5)

Ⓖ $\frac{4}{x} = \frac{x}{9}$
$\sqrt{x^2} = \sqrt{36}$
(x = 6)

(c) 3 and 16

Ⓐ $\frac{3+16}{2}$
$\frac{19}{2}$
(9.5)

Ⓖ $\frac{3}{x} = \frac{x}{16}$
$x^2 = 48$
$x = \sqrt{48}$
$4\sqrt{3}$

Example 5: If 6 is a mean proportional between 9 and a number, what is the number?
(like hw # 15)

Same!

$$\frac{9}{6} = \frac{6}{x(4)}$$ ~ or ~ $$\frac{x}{6} = \frac{6}{9}$$

geometric mean (a.k.a.: mean proportional)

$\frac{9x}{9} = \frac{36}{9}$ ⟵ Same cross-products ⟶ $9x = 36$

$x = 4$

Therefore:

$\frac{9}{6} = \frac{6}{(4)}$ and $\frac{(4)}{6} = \frac{6}{9}$

3

Example 3. Solve for x.

$$\frac{x-1}{3} = \frac{3x+2}{4}$$

$$4(x-1) \qquad 3(3x+2)$$

$$9x+6 = 12x-4$$
$$-4x \qquad -4x$$
$$5x+6 = -4$$
$$\qquad -6 \quad -6$$
$$\frac{5x}{5} = \frac{-10}{5}$$

$$\boxed{x=-2}$$

$$\frac{-2-1}{3} = \frac{3(-2)+2}{4}$$
$$\frac{-3}{3} = \frac{-6+2}{4}$$
$$-1 = -1 \checkmark$$

Example 4. If $5x = 6y$, then find $x{:}y$.

$$\frac{x}{y} = \frac{6}{5}$$

$$\boxed{6{:}5}$$

$$\frac{5x}{5} = \frac{6y}{5} \qquad \frac{x}{y} = \frac{6y}{5y} \qquad \frac{x}{y} = \frac{6}{5}$$

Example 5. If $\frac{3x+2y}{5} = \frac{4x-7y}{6}$, then find $x{:}y$.

$$\frac{x}{y} = \frac{47}{2}$$

$$\boxed{47{:}2}$$

$$5(4x-7y) = 6(3x+2y)$$
$$20x-35y = 18x +12y$$
$$-18x \qquad -18x$$
$$2x-35y = 12y$$
$$\quad +35y \quad +35y$$
$$\frac{2x}{2y} = \frac{47y}{2y} \qquad \frac{x}{y} = \frac{47}{2}$$

Example 6. Solve for x.

$$\frac{x+3}{3} = \frac{2}{x-2}$$

$$(-4)$$
$$\frac{-4+3}{3} = \frac{2}{-4-2}$$
$$\frac{-1}{3} = \frac{2}{-6}$$
$$\frac{-1}{3} = -\frac{1}{3} \checkmark$$

$$\frac{3+3}{3} = \frac{2}{3-2}$$
$$\frac{6}{3} = \frac{2}{1}$$
$$2$$

$$(x+3)(x-2) = 6$$
$$x^2-2x+3x-6 = 6$$
$$x^2+x-6 = 6$$
$$\qquad +-6 \quad +-6$$

$$x^2+x-12=0$$
$$(x+4)(x-3)=0$$

$$x=\{-4,3\}$$

★*Definition*: The <u>mean</u> of a group of numbers is the average. There are several different types of averages used in math. The one you are most familiar with is called the <u>*arithmetic mean*</u>. In this type of average, you add up all the terms and then divide by the total number of terms in the group. This type of average is often used to calculate your grade in a certain class.

The type we will be using in this chapter is called a <u>*geometric mean*</u>, which is often called a <u>*mean proportional*</u>. To find the geometric mean between 2 values, you place the values as the second and third terms in a proportion. Using x in as the first and fourth terms, you then cross-multiply and solve for x.

Example 7. Johnny received a 78 and a 91 on his first 2 algebra tests. What's Johnny's average?

Example 8. Find the geometric mean between 2 and 18.

Example 9. 15 is the mean proportional between 5 and what number?