Geometry: 7.4 – Regular Polygons

A **regular** polygon is a polygon that is Equilateral and Equiangular.

Question: For a polygon, if all the interior angles are ≅, will all the exterior angles be ≅? Yes!

1. **Always, Sometimes, or Never?** A regular quadrilateral is a square. Always
2. **Always, Sometimes, or Never?** A rhombus is regular. (if a square) Sometimes
3. **Always, Sometimes, or Never?** An equilateral triangle is regular. Always
4. **Always, Sometimes, or Never?** In a regular polygon, any interior angle and any exterior angle are supp. Always

5. For a regular octagon, find the measure of: $\boxed{n=8}$

Long way...

a) one angle

$(8-2)180 = S_i$

$(6)180$

$\boxed{1080^\circ}$

All angles ≅, so $I = \frac{1080}{8}$

$\boxed{I = 135^\circ}$

then $I = 180 - E$

$= 180 - 45$

$\boxed{= 135}$

★ Shorter!

$I + E = 180$

a) 135°

b) one exterior angle

Short way → Find this FIRST!

$S_e = 360^\circ$, so $E = \frac{360}{8}$

$\boxed{E = 45^\circ}$

b) 45°

Summary: For an equiangular polygon with n sides, exterior angle E, and interior angle I:

(Exterior ∠) $E = \frac{360}{n}$ (Interior ∠) $I = 180 - E$ ~or~ $\frac{(n-2)180}{n}$

short way! Long way...

6. Find the number of *sides* of a regular polygon if:

a) one exterior ∠ = 30°

$\frac{360}{n} = 30$ $\frac{30n}{30} = \frac{360}{30}$ $\boxed{n=12}$

a) $n = 12$

b) one exterior ∠ = 40°

$\frac{360}{n} = 40$ $\frac{40n}{40} = \frac{360}{40}$ $\boxed{n=9}$

b) $n = 9$

c) one interior ∠ = 120°

$E = 180 - I$

$= 180 - 120$

$\boxed{E = 60^\circ}$

$\frac{360}{n} = 60$ $\frac{60n}{60} = \frac{360}{60}$ $\boxed{n=6}$

c) $n = 6$

d) one int. $\angle = 160°$

$E = 180 - 160$
$E = 20$

$\frac{360}{n} = 20$ $\quad \frac{20n}{20} = \frac{360}{20}$ $\quad \boxed{n = 18}$

d) $n = 18$

e) one int. $\angle = 15°$

$E = 180 - 15$
$E = 165$

$\frac{360}{n} = 165$ $\quad \frac{165n}{165} = \frac{360}{165}$
$n = 2.\overline{18}$

e) Impossible

7. For a regular 50-gon, find the measure of one exterior and one interior angle.

$n = 50$

$E = \frac{360}{50}$
$\boxed{E = 7.2}$

$I = 180 - 7.2$
$\boxed{I = 172.8}$

one ext. $\angle =$ $7.2°$

one int. $\angle =$ $172.8°$

8. Find the # of diagonals for an equiangular polygon with an angle of 170°.

8. 594 diagonals

$I = 170$
$E = 180 - 170$
$\boxed{E = 10}$

$n = \frac{360}{10}$
$\boxed{n = 36}$

$d = \frac{36(36-3)}{2}$
$\frac{18\cancel{36}(33)}{\cancel{2}}$
594

9. Find the measure of one exterior angle of a regular polygon whose # of diagonals is sixteen less than one-third of its exterior angle sum.

9. $22.5°$

$8 \cdot 26$
$16 \cdot 13$

208
$+\boxed{13}$ $\quad -\boxed{16}$
-3

$\frac{n(n-3)}{2} = \left(\frac{360}{3}\right) - 16$

$\frac{n^2 - 3n}{2} = 120 - 16$

$\frac{n^2 - 3n}{2} = 104$

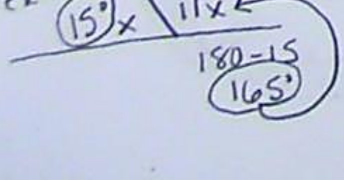
$n^2 - 3n = 208$

$n^2 - 3n - 208 = 0$
$(n+13)(n-16) = 0$
$n = \{\cancel{-13}, 16\}$

$E = \frac{360}{16} = \boxed{22.5°}$

10. In a polygon, the ratio of each interior angle to each exterior angle is 11 to 1. Give the name of this polygon.

10. 24-gon



$x + 11x = 180$
$\frac{12x}{12} = \frac{180}{12}$
$\boxed{x = 15}$

$n = \frac{360}{15}$
$\boxed{n = 24}$

Chapter 7.4 Regular Polygons

Regular Polygon (*definition*): a polygon that is both equilateral and equiangular.
In other words, all the sides are congruent and all the angles are congruent.

★ **Note:** If all the interior angles are congruent, then so are all the exterior angles. To find the measure of each exterior angle, divide 360 by the number of sides.

★ **Formula:** $E = \frac{360}{n}$

★ **Formula:** I = supplement of E or $\frac{(n-2)180}{n}$

1. What is the measure of **each exterior** angle of a regular decagon?

$\boxed{n=10}$ $E = \frac{360}{n} = \frac{360}{10} = \boxed{36^\circ}$

2. If **each exterior** angle of a polygon is 24°, then how many sides does the polygon have?

$\boxed{n=?}$ $n = \frac{360}{E} = \frac{360}{24} = \boxed{15}$ (pentadecagon)

3. In the phrase "each angle of a polygon," does the word "each" refer to an exterior or an interior angle?

"each angle" refers to the interior angles

4. What is the measure of **each** angle of a regular octagon?

$\boxed{n=8}$ $E = \frac{360}{8} = 45$ $I = 180 - E$
$= 180 - 45$
$\boxed{I = 135^\circ}$

5. The sum of the measures of the interior angles of a polygon is four times the sum of the measures of its exterior angles. How many sides does the polygon have?

$$\frac{(n-2)\cancel{180}}{\cancel{180}} = \frac{4(\overset{2}{\cancel{360}})}{\cancel{180}} \qquad n-2=8 \qquad \boxed{n=10}$$

6. The measure of **each interior** angle of a regular polygon is eight times that of an exterior angle. How many sides does the polygon have?

Let exterior angle = x

Interior angle = $8x$

$x + 8x = 180$

$9x = 180$

$x = 20$

Now: $n = 360/20$

$\boxed{n = 18}$

7. Complete the table below for each *regular* polygon named.

Number of Sides (n)	Sum of Interior Angles (S_i) $(n-2)180$	Number of Diagonals (**D**) $\frac{n(n-3)}{2}$	Sum of Exterior Angles *one per vertex* (S_e)	Each Exterior Angle (E) $\frac{360}{n}$	Each Interior Angle (I) $180 - E = I$
3	180	$\frac{3(0)}{2}$ 0	360°	$\frac{360}{3}$ 120°	180−120 60°
4	360	$\frac{4(1)}{2}$ 2	360°	$\frac{360}{4}$ 90°	180−90 90°
5	540	$\frac{5(2)}{2}$ 5	360°	$\frac{360}{5}$ 72°	180−72 108°
6	720	$\frac{6(3)}{2}$ 9	360°	$\frac{360}{6}$ 60°	180−60 120°
7	900	$\frac{7(4)}{2}$ 14	360°	$\frac{360}{7}$ $51\frac{3}{7}$°	180−$51\frac{3}{7}$ $128\frac{4}{7}$°
8	1080	$\frac{8(5)}{2}$ 20	360°	$\frac{360}{8}$ 45°	180−45 135°
9	1260	$\frac{9(6)}{2}$ 27	360°	$\frac{360}{9}$ 40°	180−40 140°
10	1440	$\frac{10(7)}{2}$ 35	360°	$\frac{360}{10}$ 36°	180−36 144°
11	1620	$\frac{11(8)}{2}$ 44	360°	$\frac{360}{11}$ $32\frac{8}{11}$°	180−$32\frac{8}{11}$ $147\frac{3}{11}$°
12	1800	$\frac{12(9)}{2}$ 54	360°	$\frac{360}{12}$ 30°	180−30 150°
15	2340	$\frac{15(12)}{2}$ 90	360°	$\frac{360}{15}$ 24°	180−24 156°
20	3240	$\frac{20(17)}{2}$ 170	360°	$\frac{360}{20}$ 18°	180−18 162°
100	17,640	$\frac{100(97)}{2}$ 4850	360°	$\frac{360}{100}$ 3.6°	180−3.6 176.4

Always!

7.4 Assign: Pp 316 – 317 (1 – 4; 7; 10 – 13)