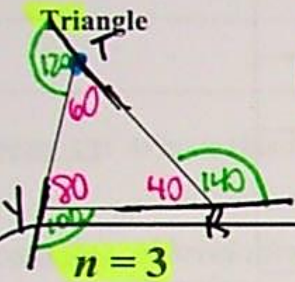
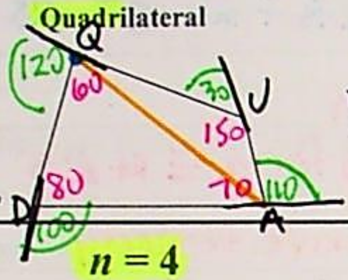
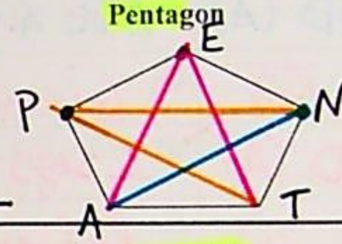
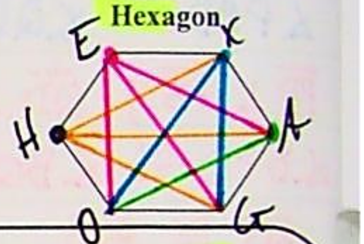
Geometry: 7.3 – Polygons and Angle Sums

# of sides	name	# of sides	name
5	Penta gon	9	Nona gon
6	Hexa gon	10	Deca gon
7	Hepta gon	12	Dodeca gon
8	Octa gon	15	Pentadeca gon

All other polygons with n sides will be called *n-gons*. **Example:** 24-gon

Triangle	Quadrilateral	Pentagon	Hexagon
			
$n = 3$	$n = 4$	$n = 5$	$n = 6$
# of Δs: 1	# of Δs: 2	# of Δs: 3	# of Δs: 4
Int. ∠ Sum: 180°	Int. ∠ Sum: 360	Int. ∠ Sum: 540°	Int. ∠ Sum: 720°
Ext. ∠ Sum: 360°	Ext. ∠ Sum: 360°	Ext. ∠ Sum: 360°	Ext. ∠ Sum: 360°
# diagonals at a vertex: 0	# diagonals at a vertex: 1	# diagonals at a vertex: 2	# diagonals at a vertex: 3
Total diag.: 0	Total diag.: 2	Total diag.: 5	Total diag.: 9

For any (convex) polygon with n sides:

Interior ∠ Sum	Exterior ∠ Sum	# of diagonals
$S_i = (n-2)180$	$S_e = 360°$ ALWAYS!	$d = \frac{n(n-3)}{2}$

G

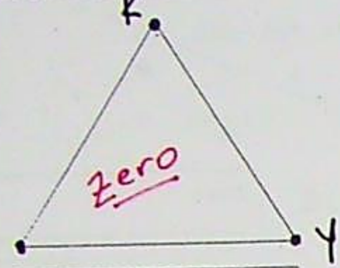
How many Diagonals? (Be Careful!)

DIRECTIONS: Draw the diagonals. Count the number of UNIQUE diagonals and record the number in the box below each figure. [Try to determine a pattern!]

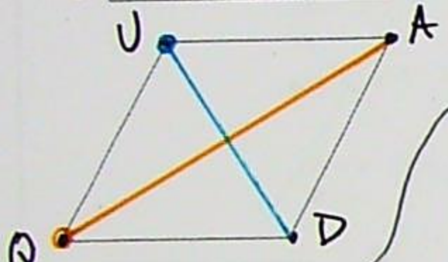
You must draw these first ⇓

sides	diagonals	
3	0	+2
4	2	+3
5	5	+4
6	9	+5
7	14	+6
8	20	+7
9	27	+8
QA10	35	

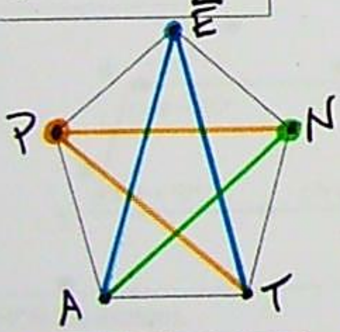
UD
~~AQ~~
~~DQ~~



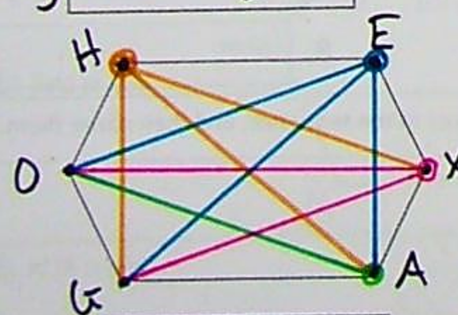
3 Triangle: 0



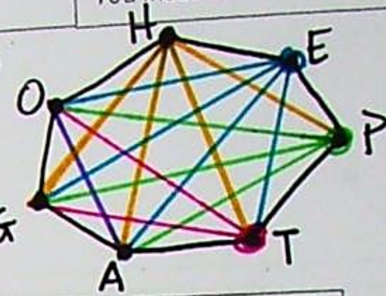
4 Quadrilateral: 2



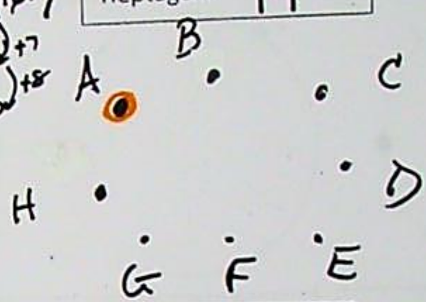
5 Pentagon: 5



6 Hexagon: 9



Heptagon: 14



8 Octagon: 20

$$\frac{\overset{4}{\cancel{8}}(5)}{\cancel{2}}$$

$$d = \frac{9(\overset{3}{\cancel{6}})}{\cancel{2}}$$

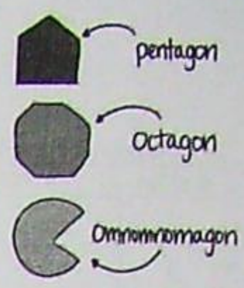
9 Nonagon: 27

$$d = \frac{n(n-3)}{2}$$

$$\frac{\overset{5}{\cancel{10}}(7)}{\cancel{2}}$$

Decagon: 35

7.3 Example Problem (Notes)



Examples Formula $S_i = (n-2)180$

1. Find the interior angle sum for a:

a) nonagon
$n=9 \quad S_i = (9-2)180$
$(7)180$
$\boxed{1260^\circ}$

b) dodecagon
$n=12$
$S_i = (12-2)180$
$(10)180$
$\boxed{1800^\circ}$

2. What is the name of the polygon whose interior angles total 2,340°?
$n = ?$

Formula $S_i = (n-2)180$

$\frac{(n-2)180}{180} = \frac{2340}{180}$
$n-2 = 13$

$\boxed{n=15 \text{ pentadecagon}}$

3. How many diagonals can be drawn in a a) heptagon? $n=7$ b) 20-gon? $n=20$

Formula $d = \frac{n(n-3)}{2}$

ⓐ $d = \frac{7(7-3)}{2} = \frac{7(\cancel{4})^{2}}{\cancel{2}} = \boxed{14}$

ⓑ $d = \frac{20(20-3)}{2} = \frac{\cancel{20}^{10}(17)}{\cancel{2}} = \boxed{170}$

4. What is the name of the polygon that has a) 54 diagonals? b) 90 diagonals?

Formula $d = \frac{n(n-3)}{2}$

ⓐ $\cancel{2}\cdot\frac{n(n-3)}{\cancel{2}} = 54 \cdot 2$ $\boxed{n=12 \text{ dodecagon}}$

$n^2 - 3n = 108$
$n^2 - 3n - 108 = 0$
$(n+9)(n-12) = 0$
$n = \{\cancel{-9}, 12\}$

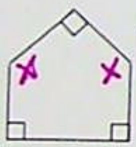
108; 9, −12; −3

ⓑ $\frac{n(n-3)}{2} = 90$ $\boxed{n=15 \text{ Pentadecagon}}$

$n^2 - 3n = 180$
$n^2 - 3n - 180 = 0$
$(n+12)(n-15) = 0$
$n = \{\cancel{-12}, 15\}$

180; −15, 12; −3

5. Three of the angles of a pentagon are right angles. The two remaining angles are congruent. Find the measure of one of the two remaining angles.



$\boxed{\text{Penta } n=5}$ $S_i = (5-2)180$
$(3)180$
$S_i = 540^\circ$

$3(90) + 2x = 540$
$270 + 2x = 540$
$\frac{2x = 270}{2}$

$\boxed{x = 135^\circ \text{ each}}$

6. Find the number of sides for a polygon with an interior angle sum of 4,820°.

$S_i = (n-2)180$

$\frac{(n-2)180}{180} = \frac{4820}{180}$
$n - 2 = 26.\overline{7}$
$n = 28.\overline{7}$ What?

Impossible! (it's not closed!)

7. Find the number of diagonals for a polygon whose interior angle sum is 8 times its exterior angle sum.

$d = \frac{n(n-3)}{2}$ $(n-2)180 = 8 * 360^\circ$

Equation: (step 1) $\frac{(n-2)1\cancel{80}}{1\cancel{80}} = \frac{8(\cancel{360}^{2})}{\cancel{180}}$
$n - 2 = 16$
$\boxed{n = 18}$

Equation: (step 2) $d = \frac{\cancel{18}^{9}(18-3)}{\cancel{2}}$
$= 9 \cdot 15$
$\boxed{= 135 \text{ diagonals}}$

Chapter 7.3 Formulas Involving Polygons

Names of Familiar Polygons

# sides	Name
3	→ Triangle
4	→ Quadrilateral
5	→ Pentagon
6	→ Hexagon
7	→ Heptagon
8	→ Octagon
9	→ Nonagon
10	→ Decagon
11	→ Undecagon
12	→ Dodecagon
15	→ Pentadecagon
n	→ n-gon

Formulas

1. sum of interior angles:
$S_i = (n-2)\cdot 180$ where n is the number of sides

2. total number of diagonals:
$$D = \frac{n(n-3)}{2}$$

3. sum of exterior angles:
$S_e = 360.$ (*always!*) Theorem #56.

Samples: $S_i = (n-2)180$

1. What is the sum of the interior angles of an $\boxed{\text{undecagon}}$? ...a $\boxed{\text{quadrilateral}}$?

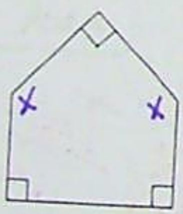
$n = 11$

$S_i = (11-2)180$
$= (9)180$
$\boxed{= 1620°}$

$n = 4$

$S_i = (4-2)180$
$= (2)180$
$\boxed{= 360°}$

2. Three of the angles of a pentagon are right angles. The two remaining angles are congruent. Find the measure of one of the two remaining angles.



Step 1: $S_i = (n-2)180$

$\boxed{n=5}$

$(5-2)180$
$(3)180$
$\boxed{S_i = 540°}$

Step 2: Equation
$2x + 3(90) = 540$
$2x + \cancel{270} = 540 - 270$
$\frac{2x = 270}{2}$
$\boxed{x = 135°}$ answer

3. What is the *name* of the polygon whose interior angles total $\boxed{2340}$? ...that total $\boxed{1080}$?

$\frac{(n-2)\cancel{180}}{\cancel{180}} = \frac{2340}{180}$

$n - \cancel{2} = 13$
$\quad +2 \quad +2$

$\boxed{n = 15 \text{ Pentadecagon}}$

$\frac{(n-2)\cancel{180}}{\cancel{180}} = \frac{1080}{180}$

$n - \cancel{2} = 6$
$\quad +2 \quad +2$

$\boxed{n = 8 \text{ Octagon}}$

Formula: $d = \frac{n(n-3)}{2}$

4. How many diagonals can be drawn in a decagon? …a heptagon?

$n=10$ $\quad$ $n=7$

$d = \frac{\cancel{10}^{5}(10-3)}{\cancel{2}}$

$= 5(7)$

$\boxed{= 35 \text{ diagonals}}$

$d = \frac{7(7-3)}{2} = \frac{7(\cancel{4}^{2})}{\cancel{2}}$

$\boxed{= 14 \text{ diagonals}}$

5. What is the name of the polygon that has 104 diagonals? …54 diagonals?

$2 \cdot \frac{n(n-3)}{2} = 104 \cdot 2$

$n^2 - 3n = 208$

$n^2 - 3n - 208 = 0$

$(n+13)(n-16) = 0$

-208: $\boxed{13}$ $\boxed{-16}$ -3

$n = \{\cancel{-13}, 16\}$

$\boxed{16\text{-gon}}$

$2 \cdot \frac{n(n-3)}{2} = 54 \cdot 2$

$n^2 - 3n = 108$

$n^2 - 3n - 108 = 0$

$(n+9)(n-12) = 0$

-108: $\boxed{9}$ $\boxed{-12}$ -3

$n = \{\cancel{-9}, 12\}$

$\boxed{\text{Dodecagon}}$

6. What is the sum of the exterior angles of a pentadecagon? …a 25-gon?

$\boxed{S_e = 360°}$ $\boxed{S_e = 360°}$

ALWAYS!

7. Complete the table using the formulas listed on the first page. See if you can observe a pattern.

Number of Sides (n)	Sum of Interior Angles (S_i) = $(n-2)180$	Number of Diagonals (D) = $\frac{n(n-3)}{2}$	Sum of Exterior Angles *one per vertex* (S_e) = 360
3 −2	1(180) = 180°	3−3 $\frac{3(0)}{2}$ $\frac{0}{2} = 0$	360°
4 −2	2(180) = 360°	4−3 $\frac{\cancel{4}^{2}(1)}{\cancel{2}}$ = 2	360°
5 −2	3(180) = 540°	5−3 $\frac{5(\cancel{2})}{\cancel{2}}$ = 5	360°
6 −2	4(180) = 720°	6−3 $\frac{\cancel{6}^{3}(3)}{\cancel{2}}$ = 9	360°
7 −2	5(180) = 900°	7−3 $\frac{7(\cancel{4}^{2})}{\cancel{2}}$ = 14	360°
8 −2	6(180) = 1080°	8−3 $\frac{\cancel{8}^{4}(5)}{\cancel{2}}$ = 20	360°
9 −2	7(180) = 1260°	9−3 $\frac{9(\cancel{6}^{3})}{\cancel{2}}$ = 27	360°
10 −2	8(180) = 1440°	10−3 $\frac{\cancel{10}^{5}(7)}{\cancel{2}}$ = 35	360°
11 −2	9(180) = 1620°	11−3 $\frac{11(\cancel{8}^{4})}{\cancel{2}}$ = 44	360°
12 −2	10(180) = 1800°	12−3 $\frac{\cancel{12}^{6}(9)}{\cancel{2}}$ = 54	360°
15 −2	13(180) = 2340°	15−3 $\frac{15(\cancel{12}^{6})}{\cancel{2}}$ = 90	360°
20 −2	18(180) = 3240°	20−3 $\frac{\cancel{20}^{10}(17)}{\cancel{2}}$ = 170	360°
100 −2	98(180) = 17,640°	100−3 $\frac{\cancel{100}^{50}(97)}{\cancel{2}}$ = 4850	360°

Always!

7·3 Assign: Pp 309 - 311 (1 - 7; 10 - 14; 17, 21)