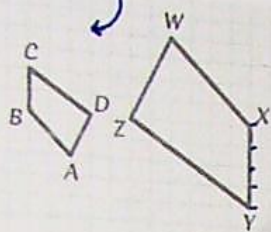


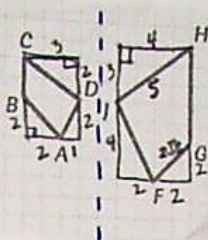
## CRITIQUE & EXPLAIN

Helena and Edwin were asked to apply a composition of transformations to ABCD, as shown here.

Helena



Edwin



### 7-2

Similarity Transformations

PearsonRealize.com

$AB = 2\sqrt{2}$      $FG = 2\sqrt{2}$   
 $BC = 2$      $GH = 5$   
 $CD = \sqrt{13}$      $HJ = 5$   
 $AD = \sqrt{5}$      $FJ = \sqrt{20}$

Sides are not proportional

A. Use Appropriate Tools Is there a composition of transformations that maps ABCD to the second figure in each student's work? If so, what is it? © MP.5

$ABCD \rightarrow WXYZ$

Rotation of  $180^\circ$

$\frac{XY}{BC} = \frac{4}{2} = 2$

Dilation,  $SF = 2$

$ABCD \rightarrow FGHI$

Reflection over vertical line

~~X~~ <sup>NO</sup> There is no composition of transformations that maps ABCD to FGHI since the sides are not proportional

B. For each student whose work shows a composition of transformations, describe the relationship between the figures.

- The side lengths of WXYZ are twice the lengths of the corresponding sides of ABCD.
- Corresponding angles are congruent (use RTDs to verify)
- The point of rotation is located halfway between vertices D & Z.

### HABITS OF MIND

Reason Given the preimage and image, how do you decide what transformations are used to create the image? © MP.2

Step 1: Compare side lengths to see if there is a relationship (ratio). If sides are different lengths, test to see if the pairs of sides share a common factor.

Step 2: Consider how rotating (turning), reflecting (flipping), or translating (sliding) the pre-image to overlay the image.



**EXAMPLE 1** Try It! Graph a Composition of a Rigid Motion and a Dilation

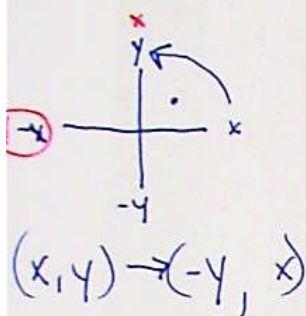
1. The vertices of  $\triangle XYZ$  are  $X(3, 5)$ ,  $Y(-1, 4)$ , and  $Z(1, 7)$ .  
 a. What is the graph of the image  $(D_2 \circ T_{(1, -2)})(\triangle XYZ)$ ?

$X(3, 5) \rightarrow (3+1, 5-2) \rightarrow (4, 3) \times 2$   
 $Y(-1, 4) \rightarrow (-1+1, 4-2) \rightarrow (0, 2) \times 2$   
 $Z(1, 7) \rightarrow (1+1, 7-2) \rightarrow (2, 5) \times 2$

$X'(8, 6)$
$Y'(0, 4)$
$Z'(4, 10)$

- b. What is the graph of the image  $(D_3 \circ r_{(90^\circ, 0)})(\triangle XYZ)$ ?

$X(3, 5) \rightarrow (-5, 3) \times 3 = X'(-15, 9)$   
 $Y(-1, 4) \rightarrow (-4, -1) \times 3 = Y'(-12, -3)$   
 $Z(1, 7) \rightarrow (-7, 1) \times 3 = Z'(-21, 3)$



**EXAMPLE 2** Try It! Describe a Composition of a Rigid Motion and a Dilation

2. If the transformations in Example 2 are performed in the reverse order, are the results the same? Do you think your answer holds for all compositions of transformations? Justify your answers.

$\triangle XYZ \rightarrow \triangle JKL$   
 $180^\circ$  rotation & Dilation  $SF=2$   
 $X(2, 1) \rightarrow (-2, -1) \rightarrow X'(-4, -2)$   
 $Y(1, 4) \rightarrow (-1, -4) \rightarrow Y'(-2, -8)$   
 $Z(4, 1) \rightarrow (-4, -1) \rightarrow Z'(-8, -2)$

$X(4, 2) \rightarrow (-4, -2) \checkmark$   
 $Y(2, 8) \rightarrow (-2, -8) \checkmark$   
 $Z(8, 2) \rightarrow (-8, -2) \checkmark$

Yes - same result

No - this reversal of the composition of transformations would not result in same outcome if  $90^\circ$  rotation or a translation is involved (This is due to how vertices are acted upon by operations involved)

**HABITS OF MIND**

**Make Sense and Persevere** Do you think there is a composition using different transformations that could produce the same image? Explain. **MP.1**

Yes - a  $180^\circ$  rotation is equivalent to a vertical and horizontal reflection

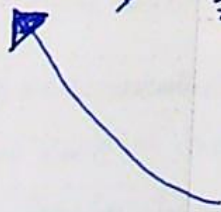
$D_2 \circ R_y \circ R_x (\triangle XYZ)$

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Dilation  $\triangle XYZ$ ,  $R_x \circ R_y$

$X(4, 2) \rightarrow (-4, -2)$   
 $Y(2, 8) \rightarrow (-2, -8)$   
 $Z(8, 2) \rightarrow (-8, -2)$

Reverse

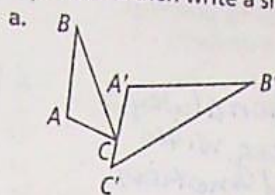




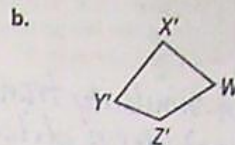
EXAMPLE 3

Try It! Find Similarity Transformations

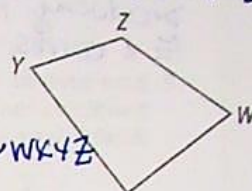
3. Describe a possible similarity transformation for each pair of similar figures shown, and then write a similarity statement.



$\Delta ABC \rightarrow \Delta A'B'C'$   
 rotation clockwise,  
 then dilation  
 $\Delta A'B'C' \sim \Delta ABC$

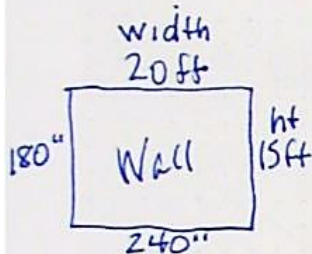


$WXYZ \rightarrow W'X'Y'Z'$



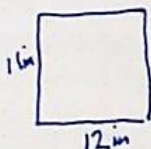
$W'X'Y'Z' \sim WXYZ$

- 180° rotation, reflection over vertical line, dilation
- reflect over horizontal, dilate



EXAMPLE 4 Try It! Determine Similarity

Convert to inches  
 $20 \cdot 12 = 240$   
 $15 \cdot 12 = 180$



4. Suppose the artist cuts 2 inches from the width of her sketch in Example 4. How much should she cut from the height so she can copy a similar image to cover the wall?

$$\frac{180}{11} \neq \frac{240}{14}$$

$16.36 \neq 17.1$

$$\frac{180}{20} = \frac{240}{x}$$

$$\frac{180}{x} = 20$$

$$20x = 180$$

$$x = 9$$

$$11 - 9 = 2 \text{ in}$$

HABITS OF MIND

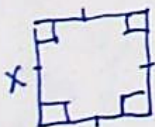
Communicate Precisely If two figures are congruent, are they also similar? If two figures are similar, are they also congruent? Explain. MP.6

- All congruent ( $\cong$ ) figures are similar - all corresponding parts (angles & sides) are congruent.
- If two figures are similar, they have the same shape ( $\cong$  angles) but sides are proportional ( $\neq$ )

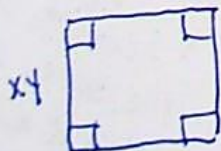
EXAMPLE 5

Try It! Identify Similar Circles

5. Write a proof that any two squares are similar.



SF:  $y$   
 SF:  $\frac{1}{y}$



Ratio of sides

$$\frac{x}{x} = \frac{x}{x} \Rightarrow 1:1$$

$$\frac{xy}{x} = \frac{x}{x} \Rightarrow y:1$$

$$\frac{x}{y} = \frac{x}{y} \Rightarrow 1:y$$

HABITS OF MIND

Make Sense and Persevere What are some other types of figures that are always similar to each other? Can you use a proof similar to the one in Example 5 to show they are always similar? Explain. MP.1.

Equilateral Triangles & Circles