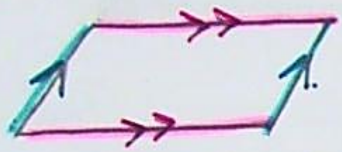
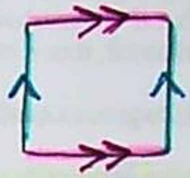
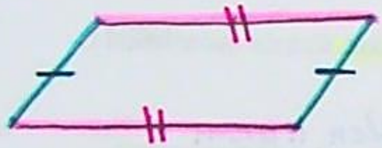
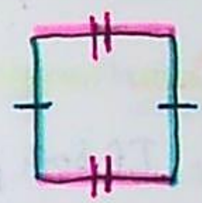
Any one of the following methods can be used to prove that a quadrilateral is a parallelogram: 5 Methods

Draw an example to illustrate the meaning of following methods of proof:

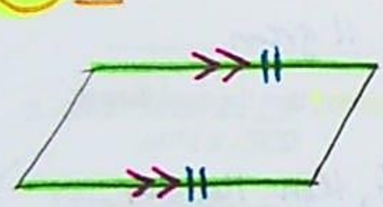
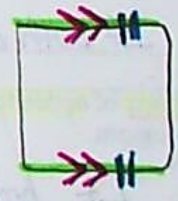
1. If both pairs of opposite sides of a quadrilateral are parallel

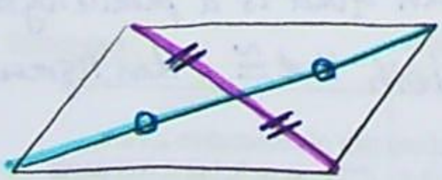
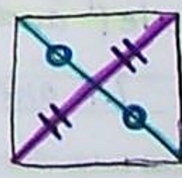
2. If both pairs of opposite sides of a quadrilateral are congruent

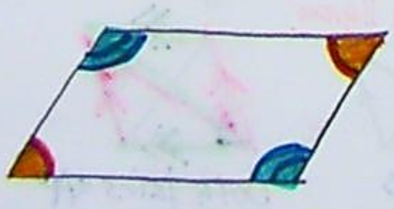
3. If one pair of opposite sides are both parallel and congruent

4. If the diagonals bisect each other

5. If both pairs of opposite angles are congruent




5.6 Notes: Proving that a Quadrilateral is a Parallelogram

5.6 Proving that a Quadrilateral is a Parallelogram

Geometry Notes

Geometry 5.6 - Proving a Quadrilateral is a Parallelogram

If we know a quadrilateral is a parallelogram, we know a number of properties about that parallelogram, specifically about its opposite sides, angles, and diagonals.

Now we are going to learn how to prove a quadrilateral is a parallelogram (▱), and it works out that if we can show that any one of the parallelogram properties apply to a given quadrilateral, then it must be a ▱.

So, the ways to prove a quad. is a ▱:

"Short way" is how to write REASONS in proofs!

1. If both pairs of opposite sides of a quadrilateral are parallel, then the quadrilateral is a parallelogram. (The converse of the definition).

Short way: If both pairs opp sides ∥, then ∥gram

2. If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

Short way: If both pairs opp sides ≅, then ∥gram

3. If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

Short way: If both diagonals bis., then ∥gram

4. If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

Short way: If both pairs opp ∠'s ≅, then ∥gram

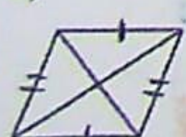
* 5. If one pair of opposite sides of a quadrilateral is both parallel and congruent, then quad is a parallelogram

Short way: If one pair opp sides both ∥ & ≅, then ∥gram

(Note: Recall that another property of ▱s is that any pair of consecutive ∠s are supp. This would actually be a way to show a quad. is a ▱ as well, but it will never be used because there would always be a shorter way to prove it.)

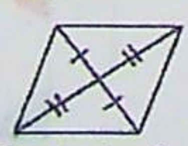
Practice: Which, if any, of the quadrilaterals below can be shown to be a ▱ ?

a)



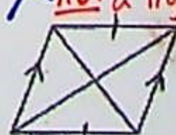
Both pairs opp sides ≅

b)



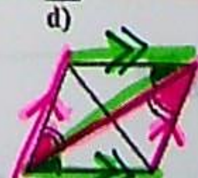
Both diagonals bis.

~~c)~~ not a ∥gram!



Isosceles Trap

d)



Both pairs opp sides ∥

5.6 Notes: Proving that a Quadrilateral is a Parallelogram

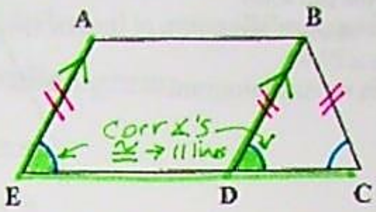
5.6 Proving that a Quadrilateral is a Parallelogram

Geometry Notes

5 Methods of Proof
- 2 prs opp sides ll
- 2 prs opp sides ≅
- 2 prs opp ∡'s ≅
- diagonals bisect
- 1 pr opp sides BOTH ≅ & ll

1. Given: $\triangle BCD$ is isosceles, with base $\overline{CD}$.
   $\overline{AE} \cong \overline{BD}$
   $\angle C \cong \angle E$

   Prove: ABDE is a ▱

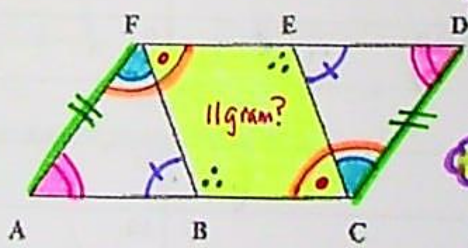


Statements	Reasons
1) $\triangle BCD$ is isosc., base $\overline{CD}$	1) Given
2) $\overline{BD} \cong \overline{BC}$	2) the legs of isosc. △'s are ≅
3) ∡BDC ≅ ∡C	3) If △, then △
4) ∡C ≅ ∡E	4) Given
5) ∡BDC ≅ ∡E	5) Transitive Property
* 6) $\overline{AE} \parallel \overline{BD}$	6) Corres ∡'s ≅ ⇒ ll lines
* 7) $\overline{AE} \cong \overline{BD}$	7) Given
8) ABDE is a ▱	8) If quad has one pair of sides both ≅ & ll, then ▱

5 Methods of Proof
- 2 prs. opp sides ll
- 2 prs opp sides ≅
- 2 prs opp ∡'s ≅
- diagonals bisect
- 1 pr. opp sides BOTH ≅ & ll

2. Given: ACDF is a ▱
   ∠AFB ≅ ∠ECD

   Prove: FBCE is a ▱



Statements	Reasons
1) ACDF is a ▱	1) Given
2) ∡AFD ≅ ∡DCA	2) In ▱, opp. ∡'s are ≅
3) ∡AFB ≅ ∡ECD (A)	3) Given
* 4) ∡BFE ≅ ∡ECB	4) Subtraction Property
5) $\overline{FA} \cong \overline{DC}$ (S)	5) In ▱, opp sides are ≅
6) ∡A ≅ ∡D (A)	6) same as #2
7) △AFB ≅ △DCE	7) ASA (3, 5, 6)
8) ∡ABF ≅ ∡DEC	8) CPCTC
9) ∡ABF supps ∡FBC ∴	9) If 2 ∡'s form str ∡, then supp
10) ∡DEC supps ∡CEF ∴	10) Same as #9
* 11) ∡FBC ≅ ∡CEF	11) Supps of ≅ ∡'s are ≅
12) FBCE is a ▱	12) If quad has both pairs opp ∡'s ≅, then ▱

35

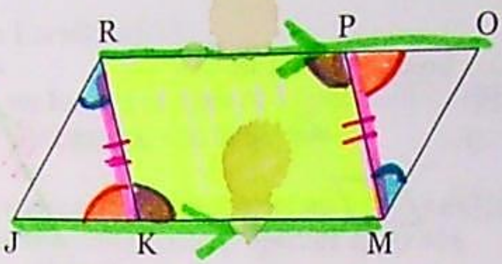
Example:

1.) (HW problem # 4 on pg. 252)

Given: RKMP is a parallelogram

$\angle JRK \cong \angle PMO$

Prove: RJMO is a parallelogram

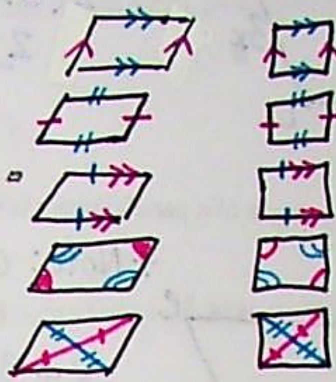


Statements	Reasons
1 RKMP is a ▱	1 Given
* 2 $\overline{RO} \parallel \overline{JM}$	2 In ‖gram, both prs. opp sides ‖
3 $\overline{RK} \cong \overline{PM}$ (S)	3 In ‖gram, both prs. opp sides ≅
4 $\angle RKM \cong \angle MPR$	4 In ‖ gram, both prs. opp ∠'s ≅
5 $\angle$ JKR supp. $\angle$RKM	5 If 2 ∠s form a str. ∠, then supp
6 $\angle$ OPM supp. $\angle$MPR	6 Same as # 5
7 $\angle JKR \cong \angle OPM$ (A)	7 Supps of ≅ ∠'s are ≅
8 $\angle JRK \cong \angle PMO$ (A)	8 Given
9 $\Delta JRK \cong \Delta OMP$	9 ASA (7, 3, 8)
10 $\overline{JK} \cong \overline{PO}$	10 CPCTC
11 $\overline{RP} \cong \overline{KM}$	11 Same as #3
* 12 $\overline{RO} \cong \overline{JM}$	12 Addition Property
13 RJMO is a ▱	13 If quad has one pr. sides BOTH ‖ & ≅, then ‖gram

5.6 Proving a Quadrilateral is a Parallelogram

5 ways to prove that a quad is a parallelogram:

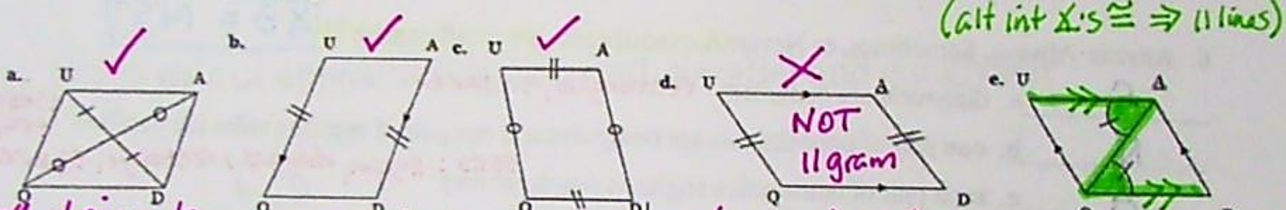
1. if both pairs of opposite sides of a quad are $\parallel \rightarrow \square$
2. if both pairs of opposite sides of a quad are $\cong \rightarrow \square$
3. if one pair of opposite sides of a quad is both $\cong / \parallel \rightarrow \square$
4. if both pairs of opposite angles of a quad are $\cong \rightarrow \square$
5. if the diagonals of a quad bisect each other $\rightarrow \square$



Problems:

1. Next to each method above, draw a diagram (or symbol) that depicts what the method is saying.
2. p. 251 #1 For each quadrilateral QUAD, state the property or definition (if there is one) that proves QUAD is a parallelogram.

(alt int ∠'s ≅ ⇒ ∥ lines)

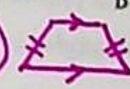


Both diagonals bisect each other

One pair sides Both ∥ & ≅

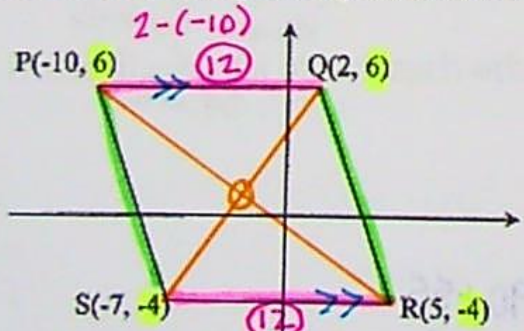
2 pairs opp sides ≅

(isos. trap)



2 pairs opp sides ∥

3. Show that PQRS is a parallelogram. (there are several ways of doing this)



$\overline{PQ} \cong \overline{SR}$

$12 = 12$

$\overline{PQ} \parallel \overline{SR}$

$m = 0$

slope $\overline{PQ}$: $\frac{6-6}{2-(-10)} = \frac{0}{12}$, $m = 0$

slope $\overline{SR}$: $\frac{-4-(-4)}{5-(-7)}$, $\frac{0}{12}$, $m = 0$

One pair of sides both ∥ & ≅, PQRS is a parallelogram

mdpt $\overline{PR}$: $\left(\frac{-10+5}{2}, \frac{6+(-4)}{2}\right)$, $\left(-\frac{5}{2}, 1\right)$

mdpt $\overline{QS}$: $\left(\frac{2+(-7)}{2}, \frac{6+(-4)}{2}\right)$, $\left(-\frac{5}{2}, 1\right)$

same mdpt

If both diagonals of quad bisect each other, then PQRS is a parallelogram

~ or ~

Two pairs of opposite sides ∥, PQRS is a parallelogram

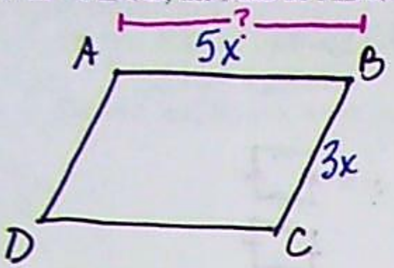
slope $\overline{PS}$: $\frac{6-(-4)}{-10-(-7)}$, $\boxed{\frac{10}{-3}}$

slope $\overline{QR}$: $\frac{6-(-4)}{2-5}$, $\boxed{\frac{10}{-3}}$

same slope

$\overline{PS} \parallel \overline{QR}$

4. In ▱ABCD, the ratio of AB to BC is 5:3. If the perimeter of ABCD is 32, find AB.



$2(AB+BC)=P$

$2(5x+3x)=32$

$\frac{2(8x)}{2}=\frac{32}{2}$

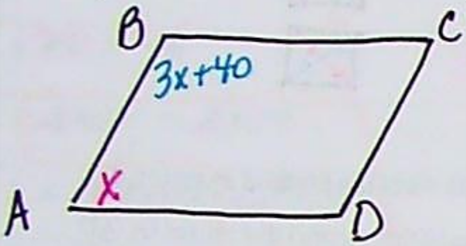
$8x=16$

$\boxed{x=2}$

$AB=5x$
$=5(2)$
$\boxed{AB=10u}$

Check
$2(10+6)=32$
$2(16)=32$
$32=32$ ✓

5. The measure of one angle of a parallelogram is 40 more than 3 times another. Find the measure of each angle.



*Note: Can't be congruent angles — must be same-side int supp!

$x+3x+40=180$

$\frac{4x}{4}=\frac{140}{4}$

$\boxed{x=35}$

$\boxed{\angle A=x=35^\circ}$

$\angle B=3x+40$
$=3(35)+40$
$=105+40$
$\boxed{\angle B=145^\circ}$

6. Answer Always, Sometimes, or Never: A quadrilateral is a parallelogram if

S a. diagonals are congruent rectangle, square

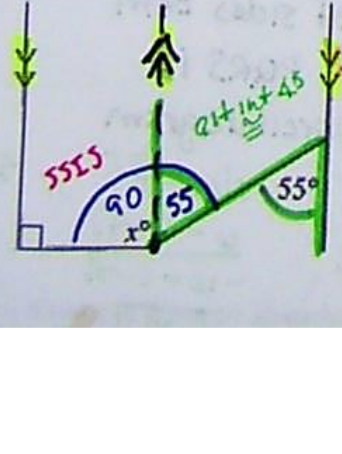
N b. one pair of opposite side are congruent and one pair of opposite sides are parallel. isosc. trap

A c. each pair of consecutive angles is supplementary SSIS: llgram, rhombus, rectangle, square

S d. all angles are right angles rectangle, square

Quad
Kite — Parallelogram — Trapezoid
Rhombus — Rectangle — Isos. Trap
Square

7. Find the value of x in the crook problem.



$x=90+55$

$\boxed{x=145^\circ}$