5.5 Notes: Properties of Quadrilaterals

5.5 Properties of Quadrilaterals — Geometry Examples

Example:

3.) (from class opener pg.249)

Given: BCDF is a kite

*BC = 3x + 4y

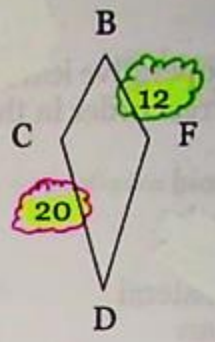
CD = 20

BF = 12

*FD = x + 2y

Find: The perimeter of BCDF

Don't really need to solve for x & y to do this one ☺



$P = 2(20u + 12u)$

$= 2(32u)$

$P = 64u$

Example:

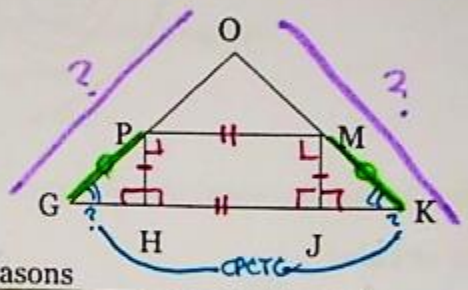
4.) (pg.246 #16)

Given: PHJM is a rectangle

$\overline{PG} \cong \overline{MK}$

Prove: ΔOGK is isosceles

HL



	Statements	Reasons
1	PHJM is a rect	1 Given
2	$\angle PHJ$ & $\angle MJH$ are Rt $\angle$'s	2 A rect has 4 Rt $\angle$'s
3	$\overline{GK} \perp \overline{PH}$ & $\overline{MJ}$	3 If Rt $\angle$s, then $\perp$ segs
* 4	$\angle PHG$ & $\angle MJK$ are Rt $\angle$'s	4 $\perp$ segs form Rt $\angle$'s
5	$\overline{PH} \cong \overline{MJ}$ (L)	5 opp sides of rect are $\cong$
6	$\overline{PG} \cong \overline{MK}$ (H)	6 Given
7	$\Delta PHG \cong \Delta MJK$	7 HL (4, 6, 5)
8	$\angle G \cong \angle K$	8 CPCTC
9	$\overline{OG} \cong \overline{OK}$	9 If $\triangle$, then $\triangle$
10.	ΔOGK is isosc.	10. If $\triangle$, then isosceles

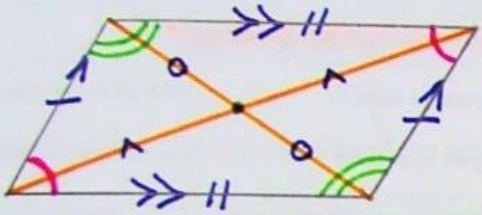
Geometry: 5.5 - Properties of Special Quadrilaterals

WARM-UP: Always, Sometimes, or Never?

1. A rhombus is a parallelogram. Always
2. A rectangle is a parallelogram. Always
3. A square is a rhombus and a rectangle. Always

Draw an example of each quadrilateral stated. Then, CIRCLE each property that applies.

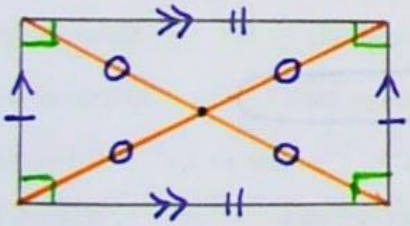
1. PARALLELOGRAM



Which properties always apply to parallelograms?

- Both pairs opposite sides ∥.
- Both pairs opposite sides ≅.
- Exactly one pair opp. sides ∥.
- Exactly one pair opp. sides ≅.
- All sides are ≅.
- Base ∠s ≅.
- Diagonals are ≅.
- Diagonals bisect each other.
- Diagonals are ⊥.
- Each pair of opp. ∠s ≅.
- Has all rt. ∠s.
- Diagonals bisect the ∠s.

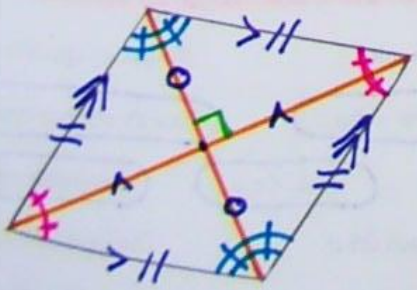
2. RECTANGLE



Which properties always apply to rectangles?

- Both pairs opposite sides ∥.
- Both pairs opposite sides ≅.
- Exactly one pair opp. sides ∥.
- Exactly one pair opp. sides ≅.
- All sides are ≅.
- Base ∠s ≅.
- Diagonals are ≅.
- Diagonals bisect each other.
- Diagonals are ⊥.
- Each pair of opp. ∠s ≅.
- Has all rt. ∠s.
- Diagonals bisect the ∠s.

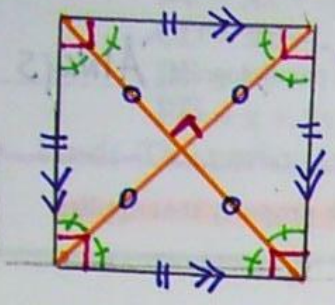
3. RHOMBUS



Which properties always apply to rhombi?

- Both pairs opposite sides ∥.
- Both pairs opposite sides ≅.
- Exactly one pair opp. sides ∥.
- Exactly one pair opp. sides ≅.
- All sides are ≅.
- Base ∠s ≅.
- Diagonals are ≅.
- Diagonals bisect each other.
- Diagonals are ⊥.
- Each pair of opp. ∠s ≅.
- Has all rt. ∠s.
- Diagonals bisect the ∠s.

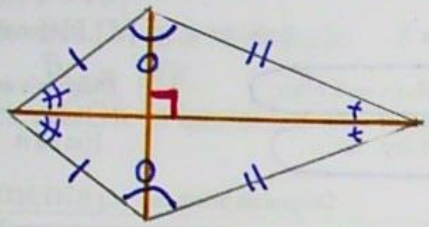
4. SQUARE



Which properties always apply to squares?

- Both pairs opposite sides ∥.
- Both pairs opposite sides ≅.
- Exactly one pair opp. sides ∥.
- Exactly one pair opp. sides ≅.
- All sides are ≅.
- Base ∠s ≅.
- Diagonals are ≅.
- Diagonals bisect each other.
- Diagonals are ⊥.
- Each pair of opp. ∠s ≅.
- Has all rt. ∠s.
- Diagonals bisect the ∠s.

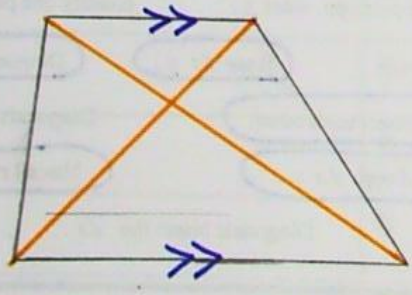
5. KITE



Which properties always apply to kites?

- Both pairs opposite sides ∥.
- Both pairs opposite sides ≅.
- Exactly one pair opp. sides ∥.
- Exactly one pair opp. sides ≅.
- All sides are ≅.
- Base ∠s ≅.
- Diagonals are ≅.
- Diagonals bisect each other.
- Diagonals are ⊥.
- Each pair of opp. ∠s ≅.
- Has all rt. ∠s.
- Diagonals bisect the ∠s.

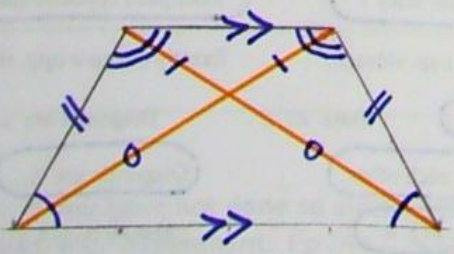
6. TRAPEZOID



Which properties always apply to trapezoids?

- Both pairs opposite sides ∥.
- Both pairs opposite sides ≅.
- Exactly one pair opp. sides ∥.
- Exactly one pair opp. sides ≅.
- All sides are ≅.
- Base ∠s ≅.
- Diagonals are ≅.
- Diagonals bisect each other.
- Diagonals are ⊥.
- Each pair of opp. ∠s ≅.
- Has all rt. ∠s.
- Diagonals bisect the ∠s.

7. ISOSCELES TRAPEZOID



Which properties always apply to isosceles trapezoids?

- Both pairs opposite sides ∥.
- Both pairs opposite sides ≅.
- Exactly one pair opp. sides ∥.
- Exactly one pair opp. sides ≅.
- All sides are ≅.
- Base ∠s ≅.
- Diagonals are ≅.
- Diagonals bisect each other.
- Diagonals are ⊥.
- Each pair of opp. ∠s ≅.
- Has all rt. ∠s.
- Diagonals bisect the ∠s.

5.5 Properties of Quadrilaterals

List all the properties of a **parallelogram** below:

1. Both pairs opposite sides parallel
2. Both pairs opposite sides congruent
3. Both pairs opposite angles congruent
4. Both diagonals bisect each other
5. One pair opposite sides are both parallel & congruent

The **rectangle** has all the properties of the parallelogram AND

1. All angles are right angles
2. Congruent diagonals

The **rhombus** has all the properties of the parallelogram AND

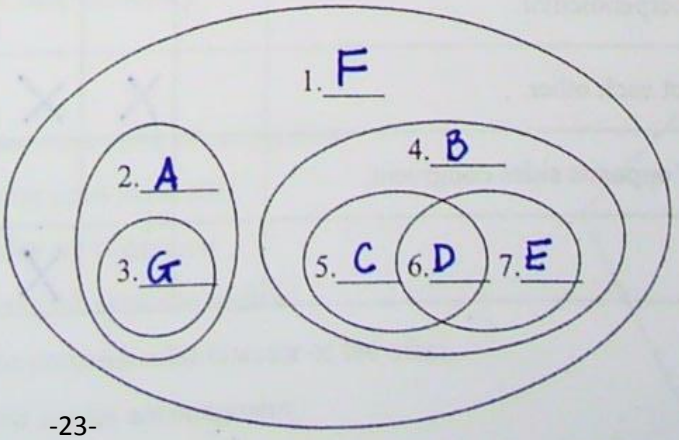
1. All sides are congruent
2. Diagonals bisect opposite angles
3. Diagonals are perpendicular

The **square** has all the properties of the parallelogram AND

1. All angles are right angles } all properties of rectangle
2. Congruent diagonals.
3. All sides are congruent } all properties of rhombus
4. Diagonals bisect opposite angles
5. Diagonals are perpendicular

Complete the Venn diagram using the letter of the word(s) provided.

A. Trapezoid
B. Parallelogram
C. Rhombus
D. Square
E. Rectangle
F. Quadrilateral
G. Isosceles Trapezoid



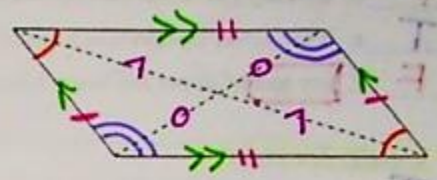
5.5 Quadrilateral Properties Matrix

Property	Quadrilateral	Parallelogram	Rectangle	Rhombus	Square	Trapezoid	Isosceles Trapezoid	Kite
Both pairs of opposite sides are congruent.		X	X	X	X			
Both pairs of opposite angles are congruent.		X	X	X	X			
Has 4 sides.	X	X	X	X	X	X	X	X
Has 4 angles.	X	X	X	X	X	X	X	X
Both diagonals are congruent.			X		X		X	
4 congruent sides.				X	X			
4 congruent angles.			X		X			
The diagonals bisect the opposite angles.				X	X			
All consecutive angles are supplementary.		X	X	X	X			
Both pairs of opposite sides are parallel.		X	X	X	X			
Exactly one pair of opposite sides are parallel.						X	X	
The diagonals are perpendicular.				X	X			X
The diagonals bisect each other.		X	X	X	X			
Exactly one pair of opposite sides congruent.							X	
4 right angles.			X		X			

5.5 Properties of Quadrilaterals – True or False

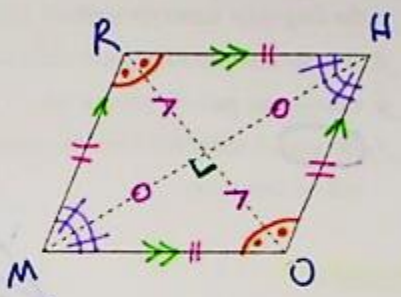
Mark each as **true** or **false**. Remember, that a property is true only if it is true all the time.

1. *Parallelogram*



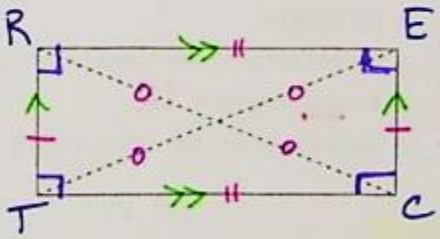
T a. both pairs of opposite sides are parallel
T b. both pairs of opposite sides are congruent
T c. both pairs of opposite angles are congruent
T d. all consecutive angles are supplementary
F e. the diagonals are congruent
T f. the diagonals bisect each other

2. *Rhombus*



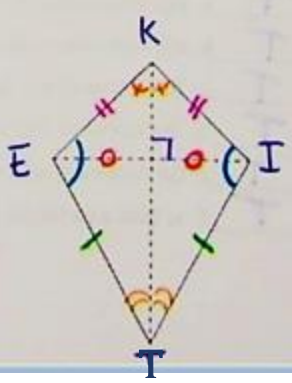
T a. all consecutive angles are supplementary
T b. all consecutive sides are congruent
F c. all consecutive sides form right angles
T d. the diagonals are perpendicular
T e. the diagonals bisect the opposite angles
F f. exactly one pair of opposite sides is parallel

3. *Rectangle*



F a. the diagonals are perpendicular
T b. the diagonals are congruent
F c. the diagonals bisect the opposite angles
T d. the diagonals bisect each other
T e. all consecutive angles are congruent
T f. all consecutive angles are supplementary

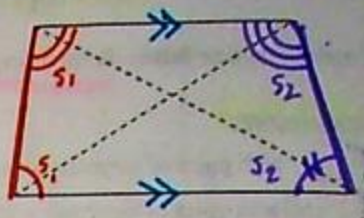
4. *Kite*



T a. two disjoint pairs of consecutive sides are congruent
F b. exactly one pair of opposite sides is parallel
F c. both pairs of opposite sides are congruent
T d. at least one diagonal bisects the opposite angles
T e. at least one diagonal is the perpendicular bisector of the other
T f. at least one pair of opposite angles are congruent

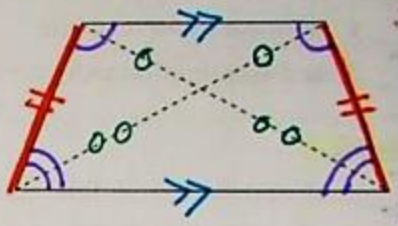
5.5 Notes: "Properties of Quadrilaterals"

5. *Trapezoid*



F a. both pairs of opposite sides are parallel
F b. the diagonals are congruent
F c. the diagonal bisect each other
F d. the diagonal bisect the opposite angles
T e. it can never be drawn as a concave polygon
F f. any pair of upper and lower base angles will be supplementary

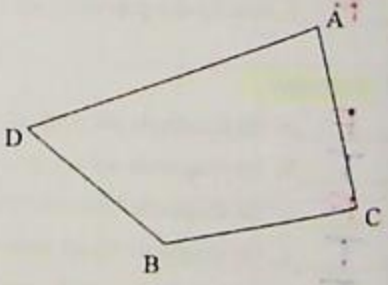
6. *Isosceles Trapezoid*



T a. the diagonals are congruent
F b. the diagonals bisect each other
F c. the diagonals bisect the opposite angles
T d. exactly one pair of opposite sides are parallel
T e. exactly one pair of opposite sides is congruent
T f. any pair of upper and lower base angles will be supplementary and the upper and lower base angles will be congruent.

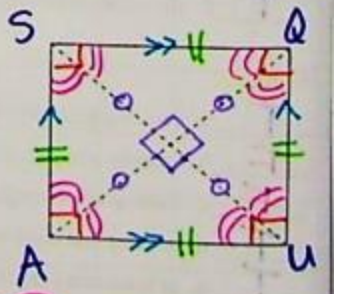
7. *Quadrilateral*

F a. has 3 sides
F b. has 5 angles
T c. can be drawn as either a convex or a concave polygon
F d. has four diagonals (2)
F e. the one shown could be named ABCD



8. *Square*

T a. any two consecutive sides are congruent
T b. any two consecutive angles are supplementary
T c. the slopes of the diagonals are opposite reciprocals
T d. the slopes of any two consecutive sides are opposite reciprocals
T e. the diagonals split opposite angles into two 45° angles.
T f. is both a rectangle and a rhombus at the same time



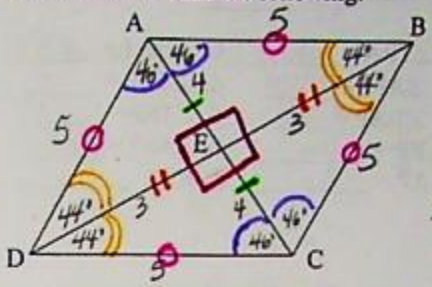
The Maserati of Quads!
"Really Fancy"

5.5 Notes: "Properties of Quadrilaterals"

5.5 Special Quads #1

1. Given: ABCD is a *rhombus*. $\angle ABC = 88°$, AD = 5, AE = 4, and DB = 6. Find the following:

a. $\angle ABD =$ 44° b. $\angle DAC =$ 46°
c. $\angle ADC =$ 88° d. $\angle AED =$ 90°
e. $\angle BCD =$ 92° f. DC = 5u
g. AC = 8u h. DE = 3u

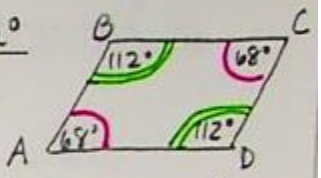


$\frac{180}{-88}$
$\frac{92}{2} = 46$

2. In □ABCD, $\angle A = 68°$. Find the measure of the other angles. Draw a sketch to help you.

a. $\angle B =$ 112° b. $\angle C =$ 68° c. $\angle D =$ 112°

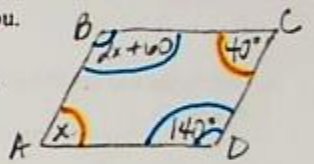
$\frac{180}{-68}$
112



3. In □ABCD, $\angle A = x$, $\angle B = 2x + 60$. $\angle D =$ 140°. Draw a sketch to help you.

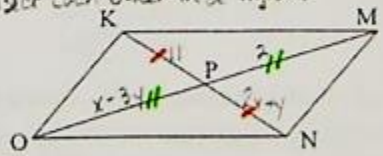
$x + 2x + 60 = 180 - 60$
$\frac{3x}{3} = \frac{120}{3}$
$x = 40$

$2(40) + 60$
$80 + 60$
$140°$ ✓



4. Given □KMNO, KP = 11, OP = $x - 3y$, PN = $2x + y$, PM = 2. Find the value of x, y, and the length of both diagonals. diagonals bisect each other in a llgram

a. $x =$ 5
b. $y =$ 1
c. OM = 4
d. KN = 22
↳ how could these be done without the equations?



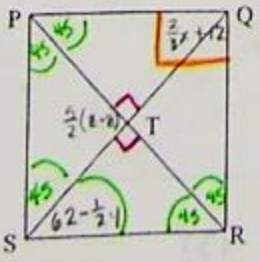
$x - 3y = 2 \Rightarrow x = 3y + 2$
$2x + y = 11$
$2(3y+2) + y = 11$
$6y + 4 + y = 11$
$7y + 4 = 11$
$7y = 7$
$y = 1$

$x = 3(1) + 2$
$x = 3 + 2$
$x = 5$

5. Given: PQRS is a *square*. $\angle PQR = \left(\frac{2}{3}x + 12\right)°$, $\angle QSR = \left(62 - \frac{1}{2}y\right)°$, and $\angle STP = \frac{5}{2}(z - 8)°$.

Find the values of x, y, and z.

a. $x =$ 117 b. $y =$ 34 c. $z =$ 44



$\frac{2}{3}x + 12 = 90 - 12$
$\frac{3}{2} \cdot \frac{2}{3}x = 78 \cdot \frac{3}{2}$
$x = 117$

$62 - \frac{1}{2}y = 45$
$-62 \quad -62$
$-2\left(-\frac{1}{2}y\right) = (-17)(-2)$
$y = 34$

$\frac{3}{5} \cdot \frac{5}{2}(z - 8) = 90 \cdot \frac{2}{5}$
$z - 8 = 36$
$+8 \quad +8$
$z = 44$

39
3
117

5.5 Special Quads #2

1. Given: TRAP is an *isosceles trapezoid*. $\angle RTP = 70°$, $AS = 4$, and $SP = 7$. Find the following:

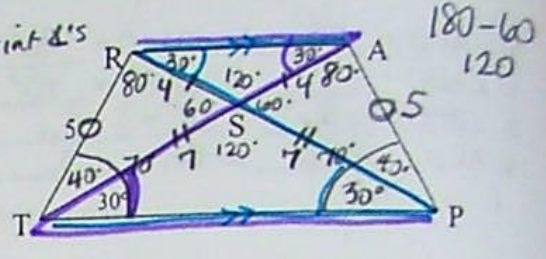
80+30 a. $\angle TRA =$ 110° b. $\angle RAT =$ 30° 11 lines → alt int ∠'s

40+30 c. $\angle APT =$ 70° d. $\angle TRP =$ 80° 110−30

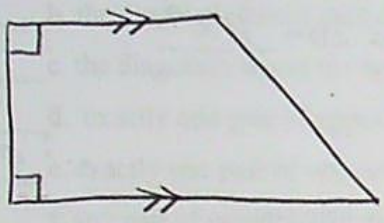
180−70 e. $\angle RAP =$ 110° f. $\angle TSP =$ 120° 180−60

4+7 g. $RP =$ 11 h. $\angle ASP =$ 60° 180−120

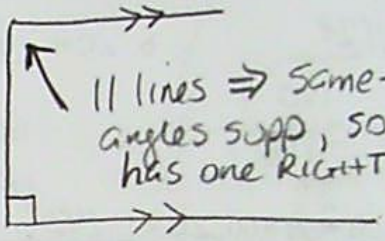
i. $AP =$ 5 j. $RS =$ 4



2. Draw a trapezoid with 2 right angles.

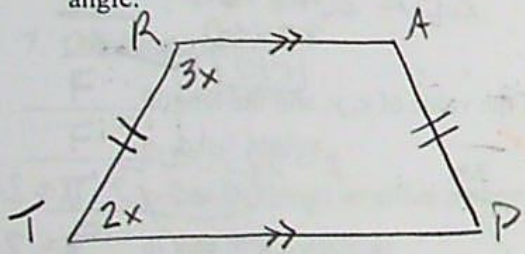


3. Draw a trapezoid with 1 right angle.



11 lines ⇒ same-side interior angles supp, so if a trapezoid has one Right ∠, it MUST have TWO!

4. The upper and lower base angles of an isosceles trapezoid are in the ratio 2:3. Find the measure of each angle.



$2x + 3x = 180$

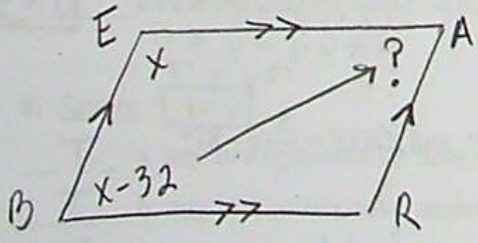
$\frac{5x}{5} = \frac{180}{5}$

$x = 36$

$\angle T = 2(36) = 72°$

$\angle R = 3(36) = 108°$

5. In $\square BEAR$, $\angle B$ is 32° less than $\angle E$. Find the measure of $\angle A$. Draw a diagram.



$x + x - 32 = 180$

$\frac{2x}{2} = \frac{212}{2}$

$x = 106$

$\angle A = x - 32$

$106 - 32$

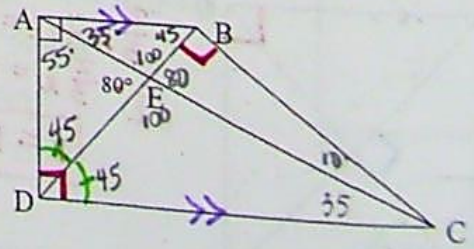
$\angle A = 74°$

6. Given: ABCD is a trapezoid with bases $\overline{AB}$ and $\overline{DC}$. $\overrightarrow{DB}$ bisects $\angle ADC$, $\angle AED = 80°$, and $\overline{DB} \perp \overline{BC}$. Find the following:

a. $\angle BDC =$ 45° b. $\angle DAE =$ 55°

c. $\angle ACD =$ 35° d. $\angle ACB =$ 10°

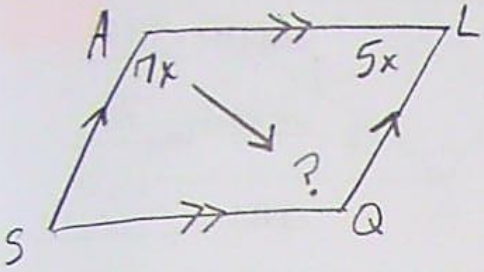
e. $\angle ABC =$ 135° f. $\angle BCD =$ 45°



5.5 Notes: Properties of Quadrilaterals

Geometry: 5.5 – Special Quad Properties #3 **Name** _____

1. SALQ is a ▱. If the ratio of ∠A to ∠L is 7 to 5, find the measure of ∠Q.



$7x+5x=180$

$\frac{12x=180}{12}$

$x=15$

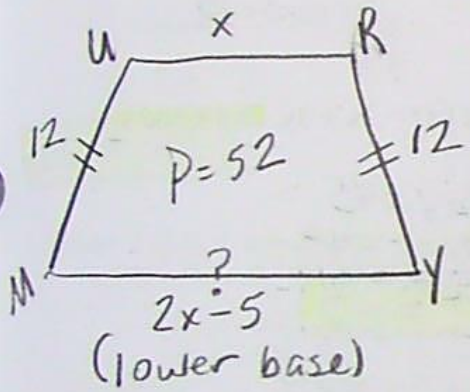
$\angle Q \cong \angle L$

$=7x$

$=7(15)$

$=105°$

2. MURY is an isosceles trapezoid with a leg of 12. If the lower base is 5 less than twice as much as the upper base, and the perimeter is 52. Find the length of the lower base.



$x+2x-5+2(12)=52$

$3x-5+24=52$

$3x+19=52$

$\frac{3x=33}{3}$

$x=11$

$\overline{MY}$

lower base

$2x-5$

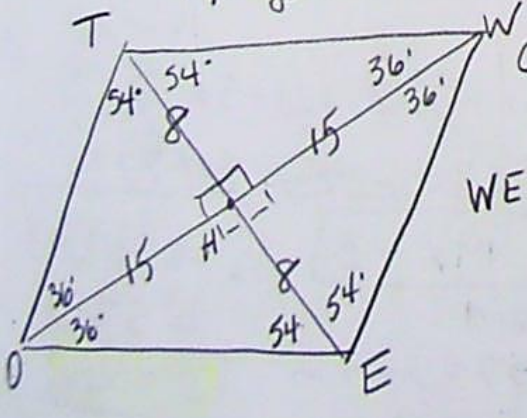
$2(11)-5$

$22-5$

$17u$

3. TWEO is a rhombus with its diagonals intersecting at H. If the perimeter of TWEO is 68, WH = 15, TE = 16, and ∠OTE = 54°, find the following. **Set up a good diagram!!!**

Pythagorean Thm: $c^2=a^2+b^2$



$c=\sqrt{a^2+b^2}$

$WE=\sqrt{8^2+15^2}$

$\sqrt{64+225}$

$\sqrt{289}$

17

∠WTE 54° ∠TEW 54°

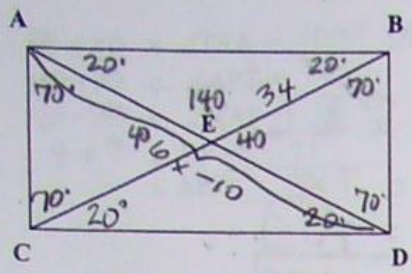
∠EOW 36° ∠TWE 72°

∠WEO 108° ∠WHE 90°

WE 17u TH 8u

WO 30u TW 17u

4. ABDC is a rectangle, with $\angle BCD = 20°$, BE = 34, and AD = $(6x - 10)$.



$6x - 10 = 2(34)$

$6x - 10 = 68$

$\frac{6x}{6} = \frac{78}{6}$

$x = 13$

$\angle EDC =$ 20°

$\angle CED =$ 140°

$\angle CAE =$ 70°

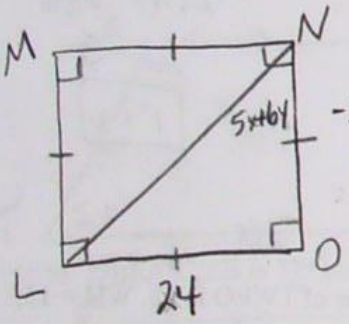
$\angle EBA =$ 20°

$\angle AEC =$ 40°

AE = 34

$x =$ 13

5. LMNO is a square with perimeter 96. If $\angle LNO = (5x + 6y)°$ and LO = 2x + 3y, find x and y.



$5x + 6y = 45$

$-2(2x + 3y = 24)$

$\frac{4s}{4} = \frac{96}{4}$

$s = 24$

$5x + 6y = 45$

$\oplus\ -4x - 6y = -48$

$x = -3$

$2(-3) + 3y = 24$

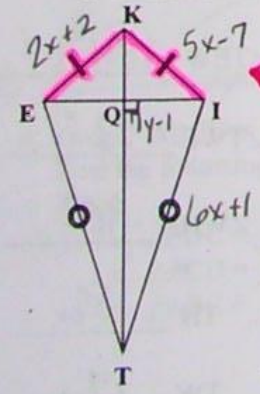
$-6 + 3y = 24$

$3y = 30$

$y = 10$

6. KITE is a kite (imaginative name, huh?). EK = 2x + 2, IT = 6x + 1, KI = 5x – 7, and $\angle IQT = (7y - 1)°$.

Find y and the perimeter of KITE.



$7y - 1 = 90$

$\frac{7y}{7} = \frac{91}{7}$

$y = 13$

$P = 2(6x+1) + (2x+2) + 5x - 7$

$12x + 2 + 2x + 2 + 5x - 7$

$P = 19x - 3$

and

Find x: $5x - 7 = 2x + 2$

$3x = 9$

$x = 3$

$\therefore P = 19(3) - 3$

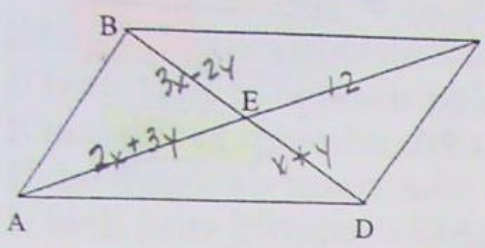
$= 57 - 3$

$P = 54$

Perimeter depends on the value of x!

Quadrilaterals and Systems of Equations

1. Given: $\square ABCD$; $BE = 3x - 2y$; $CE = 12$, $DE = x + y$; $AE = 2x + 3y$.
How long is diagonal BD?



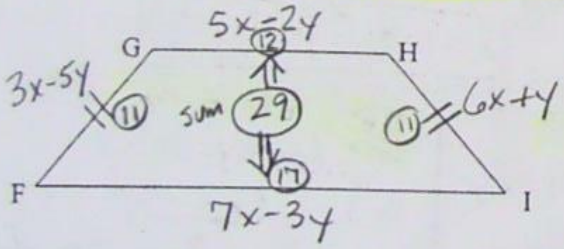
$3x-2y = x+y$
$2x-3y=0$

$2x+3y=12$
$+\ 2x-3y=0$
$4x = 12$
$\frac{4x}{4} = \frac{12}{4}$
$x=3$

$2(3)+3y=12$
$6+3y=12$
$\frac{3y=6}{3}$
$y=2$

$BD = 3x-2y+x+y$
$= 4x-y$
$= 4(3)-(2)$
$= 12-2$
$BD = 10$

2. Given: FGHI is an isosceles trapezoid; the sum of its bases is 29; $FG = 3x - 5y$; $GH = 5x - 2y$; $HI = 6x + y$; $IF = 7x - 3y$. What is the perimeter of the isosceles trapezoid?



$6x+y = 3x-5y$
$\frac{3x=-6y}{3}$
$x = -2y$

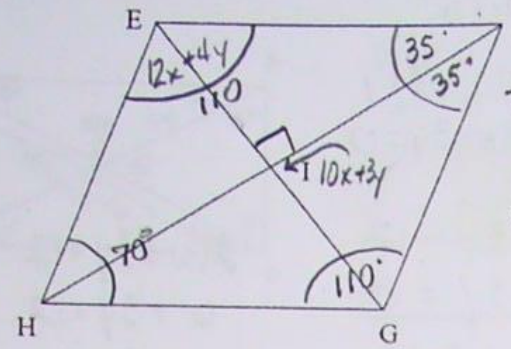
$P = 5x-2y + 6x+y + 7x-3y + 3x-5y$
$21x-9y$
$21(2)-9(-1)$
$42+9$
$P = 51u$

$29+22=51$ ✓

$5x-2y+7x-3y=29$
$12x-5y=29$
$12(-2y)-5y=29$
$-24y-5y=29$
$\frac{-29y=29}{-29}$
$y=-1$

$x=-2y$
$=-2(-1)$
$x=2$

3. Given: rhombus EFGH; $m\angle EFI = 35$; $m\angle FIG = 10x + 3y$; $m\angle FEH = 12x + 4y$.
What is the positive difference between x and y?



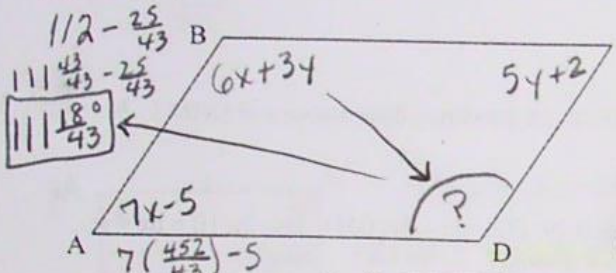
$4(10x+3y=90)$
$-3(12x+4y=110)$ ⊕

$40x+12y=360$
$-36x-12y=-330$
$4x = 30$
$x = 7.5$

$10x+3y=90$
$10(7.5)+3y=90$
$75+3y=90$
$3y=15$
$y=5$

$7.5 - 5.0 = 2.5$ difference

4. Given: $\square ABCD$; $m\angle A = 7x - 5$; $m\angle B = 6x + 3y$; $m\angle C = 5y + 2$.
Find $m\angle D$.



$180 - 68\frac{25}{43}$
$112 - \frac{25}{43}$
$111\frac{43}{43} - \frac{25}{43}$
$111\frac{18}{43}°$

$7(\frac{452}{43}) - 5$
$\frac{3164}{43} - \frac{215}{43} = \frac{2949}{43} = 68\frac{25}{43}$

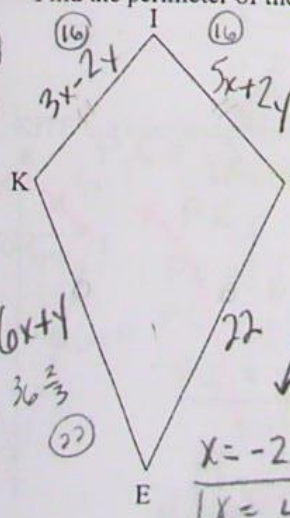
$6x+3y+7x-5=180$
(5) $13x+3y=185$

$7x-5=5y+2$
(3) $7x-5y=7$

$6x+3y+5y+2=180$
$6x+8y=178$
or $3x+4y=89$

$65x+15y=925$
⊕ $21x-15y=21$
$86x = 946$
$x = 10\frac{44}{86} = 10\frac{22}{43}(\frac{452}{43})$

6. Given: KITE is a kite; $KI = 3x - 2y$; $IT = 5x + 2y$; $TE = 22$; $EK = 6x + y$.
Find the perimeter of the kite.



$3(7\frac{1}{3}) - 2(-7\frac{1}{3})$
$22 + 14\frac{2}{3}$
$36\frac{2}{3}$

$\frac{2}{3}(\frac{22}{3}) + (-\frac{22}{3})$
$44 - 7\frac{1}{3}$
$43\frac{3}{3}$ $36\frac{2}{3}$

If: $\overline{IT} \cong \overline{IK}$
$5x+2y = 3x-2y$
$2x = -4y$
$x = -2y$

and: $\overline{EK} \cong \overline{TE}$
$6x+y=22$
$6(-2y)+y=22$
$-12y+y=22$
$-11y=22$
$y=-2$

$x = -2(-2)$
$x = 4$

$P = 2(16) + 2(22) = 76$
$32 + 44$

If: $\overline{EK} \cong \overline{KI}$
$6x+y = 3x-2y$
$3x = -3y$
$x = -y$

and: $\overline{IT} \cong \overline{TE}$
$5x+2y=22$
$5(-y)+2y=22$
$-5y+2y=22$
$-3y=22$
$y = -7\frac{1}{3}$

$x = -(-7\frac{1}{3})$
$x = 7\frac{1}{3}$

$P = 2(36\frac{2}{3}) + 2(22)$
$73\frac{1}{3} + 44$
$P = 117\frac{1}{3}$