3.8 Notes: "The HL-Postulate"

Geometry Worksheet 3.8 DPT — HL-Postulate — Date:

Postulate: If there exists a correspondence between the vertices of 2 rights triangles such that the hypotenuse and a leg of one triangle are congruent to the corresponding parts of the other triangle, the 2 right triangles are congruent by **HL.**

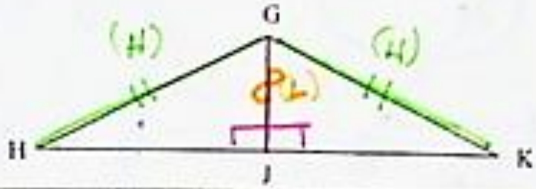
To use **HL**, we must show in our proof:

- 2 right angles, *
- 2 corresponding hypotenuses and ≅
- 2 corresponding legs ≅

*If using HL, then you do NOT have to say the right ∠'s are ≅

1. Given: $\overline{GJ}$ is an altitude to $\overline{HK}$
   $\overline{HG} \cong \overline{KG}$ (H)

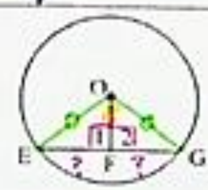
   Prove: △HGJ ≅ △KGJ



Statements	Reasons
1. $\overline{GJ}$ is an altitude to $\overline{HK}$	1. Given
*2. ∡GJH & ∡GJK are Rt ∡s	2. An alt forms Rt ∡'s on opp side of △
3. $\overline{HG} \cong \overline{KG}$ (H)	3. Given
4. $\overline{GJ} \cong \overline{GJ}$ (L)	4. Reflexive Prop
5. △HGJ ≅ △KGJ	5. HL (2; 3, 4)

2. Given: ⊙O
   $\overline{OF}$ is an altitude

   Prove: $\overline{EF} \cong \overline{FG}$



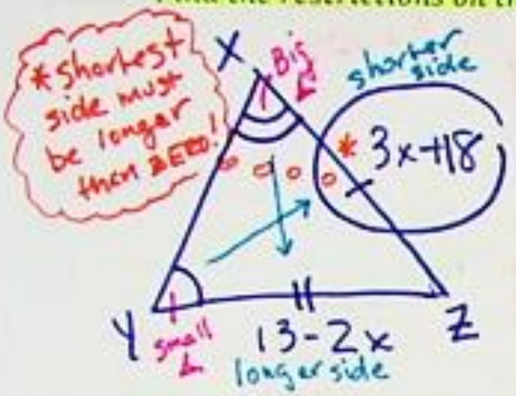
Statements	Reasons
1. ⊙O	1. Given
2. Draw $\overline{OE}$ & $\overline{OG}$	2. Any 2 points determine a line.
3. $\overline{OE} \cong \overline{OG}$ (H)	3. All radii of a ⊙ are ≅
4. $\overline{OF}$ is an altitude	4. Given
*5. ∡1 & ∡2 are Rt ∡'s	5. An alt forms Rt ∡s on opp side of △
6. $\overline{OF} \cong \overline{OF}$ (L)	6. Reflexive
7. △OEF ≅ △OGF	7. HL (5; 3, 6)
8. $\overline{EF} \cong \overline{FG}$	8. CPCTC

Class Examples:

Example 1) Draw! (and label)

Given: ΔXYZ; $(m\angle X) > (m\angle Y)$; $YZ = 13 - 2x$, and $XZ = 3x + 18$ (larger)

Find the restrictions on the value of x.



$(YZ > XZ)$

$13 - 2x > 3x + 18$

$+2x \quad +2x$

$13 > 5x + 18$

$-18 \quad -18$

$-5 > 5x$

$\frac{-5}{5} > \frac{5x}{5}$

$-1 > x \quad \underline{x < -1}$

(shorter side > 0)

$*\ 3x + 18 > 0$

$\frac{3x}{3} > \frac{-18}{3}$

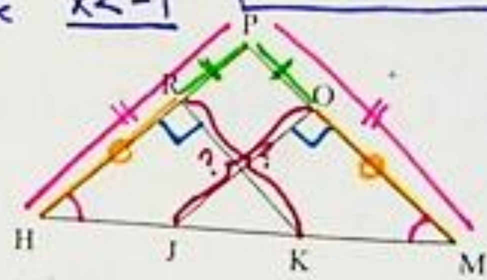
$\underline{x > -6}$

$\boxed{-6 < x < -1}$

Example 2) Proof hw problem # 9 from pg. 159

Given: $\overline{RK} \perp \overline{HR}$,
$\overline{JO} \perp \overline{PM}$,
$\overline{PR} \cong \overline{PO}$, > Subtract
$\overline{PH} \cong \overline{PM}$

Conclusion: $\overline{RK} \cong \overline{JO}$



Statements		Reasons
1. $\overline{RK} \perp \overline{HR}$		1. Given
* 2. ∠HRK is a Rt ∠		2. ⊥ segs form Rt ∠'s
3. $\overline{JO} \perp \overline{PM}$		3. Given
* 4. ∠MOJ is a Rt ∠		4. Same as #2
5. ∠HRK ≅ ∠MOJ	(A)	5. All Rt ∠'s are ≅
6. $\overline{PR} \cong \overline{PO}$		6. Given
7. $\overline{PH} \cong \overline{PM}$		7. Given
8. $\overline{RH} \cong \overline{OM}$	(S)	8. Subtraction Prop (steps 6, 7)
9. ∠H ≅ ∠M	(A)	9. If △s, then ∠s
10. △HRK ≅ △MOJ		10. ASA (5, 8, 9)
11. $\overline{RK} \cong \overline{JO}$		11. CPCTC

3.8 The HL Postulate

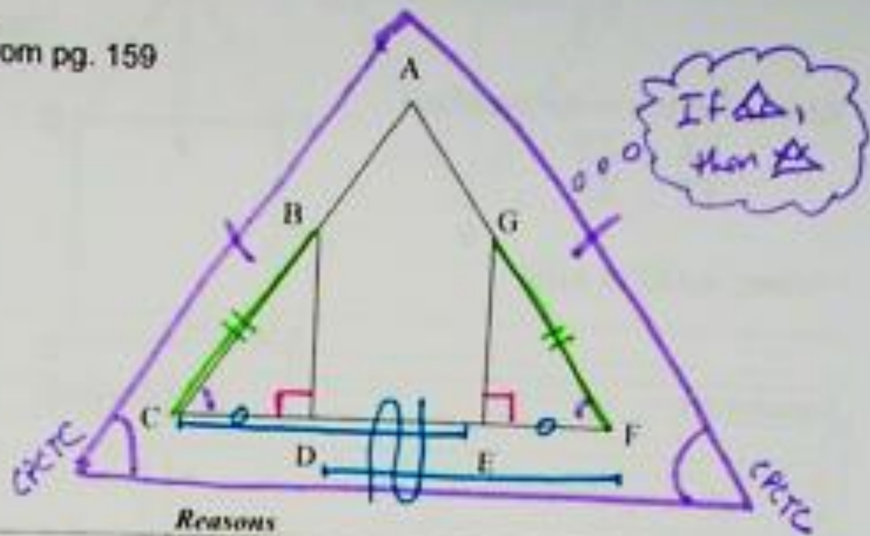
Geometry Examples

Example 3) Proof hw problem #8 from pg. 159

Given: $\overline{BD} \perp \overline{CF}$,
$\overline{GE} \perp \overline{CF}$,
$\overline{CE} \cong \overline{DF}$,
$\overline{BC} \cong \overline{GF}$

Prove: ΔACF *is isosceles.*

A △ with at least 2 ≅ sides



Statements		*Reasons*
1. $\overline{BD} \perp \overline{CF}$		1. Given
2. * △BDC is Rt △		2. ⊥ segs form Rt △s
3. $\overline{GE} \perp \overline{CF}$		3. Given
4. * △GEF is Rt △		4. Same as #2
5. $\overline{CE} \cong \overline{DF}$		5. Given
6. $\overline{DE} \cong \overline{DE}$		6. Reflexive Prop
7. $\overline{CD} \cong \overline{EF}$	(L)	7. Subtraction (steps 5 & 6)
8. $\overline{BC} \cong \overline{GF}$	(H)	8. Given
9. △BDC ≅ △GEF		9. HL (2,4; 8,7)
10. ∡C ≅ ∡F		10. CPCTC
11. $\overline{AC} \cong \overline{AF}$		11. If △, then △
12. △ACF is isos.		12. If at least 2 sides of a △ are ≅, then isosceles