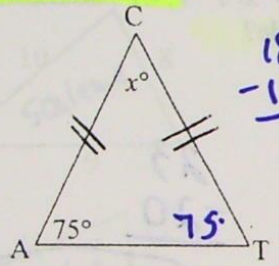
3.7 Notes: "Angle-Side Theorems"

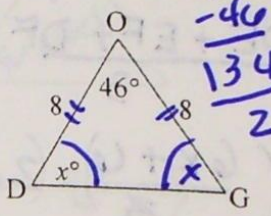
If △, then △ and If △, then △

Geometry 3.7: The Isosceles Triangle Theorem/Converse.
Find the value of x.



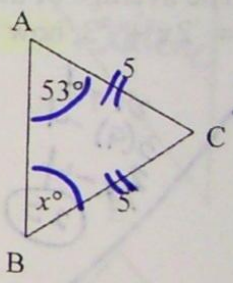
180 − 150 = 30

1. $x = 30$

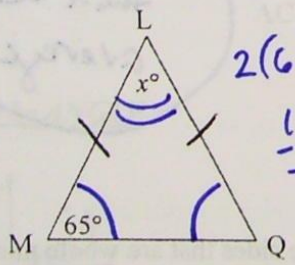


180 − 46 = 134; 134 / 2 = 67

2. $x = 67$

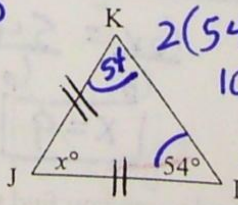


3. $x = 53$



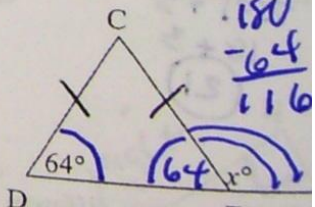
2(65) = 130; 180 − 130 = 50

4. $x = 50$



2(54) = 108; 180 − 108 = 72

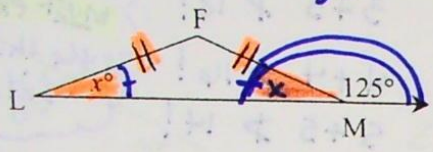
5. $x = 72$



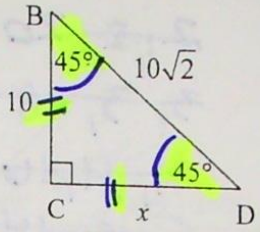
180 − 64 = 116

6. $x = 116$

180 − 125 = 55

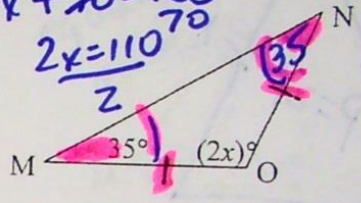


7. $x = 55$

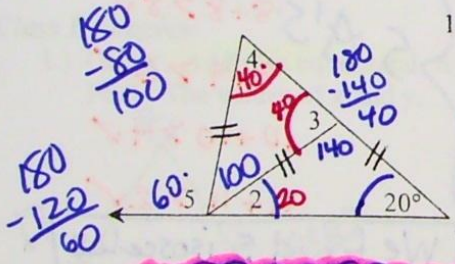


8. $x = 10$

$2x + 70 = 180$; $2x = 110$; $x = 55$



9. $x = 55$

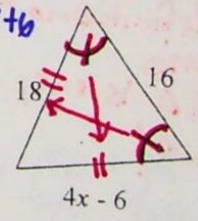


180 − 80 = 100; 180 − 140 = 40; 180 − 120 = 60

10. $\angle 2 = 20$
$\angle 3 = 40$
$\angle 4 = 40$
$\angle 5 = 60$

Note: figure not drawn to scale.

$4x - 6 = 18 + 6$; $4x = 24$; $x = 6$

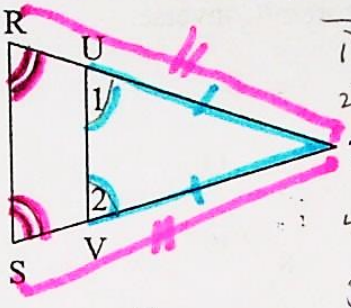


11. $x = 6$

Proofs using the Isosceles Triangle Theorem

1. Given: $\angle R \cong \angle S$
$\angle 1 \cong \angle 2$

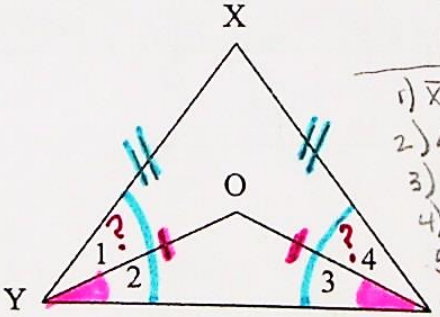
Prove: $\overline{RU} \cong \overline{SV}$



Statements	Reasons
1) $\triangle R \cong \triangle S$	1) Given
2) $\overline{RT} \cong \overline{ST}$	2) If $\triangle$, then $\triangle$
3) $\triangle 1 \cong \triangle 2$	3) Given
4) $\overline{UT} \cong \overline{VT}$	4) Same as #2
5) $\overline{RU} \cong \overline{SV}$	5) Subtraction

2. Given: $\overline{XY} \cong \overline{XZ}$
$\overline{OY} \cong \overline{OZ}$

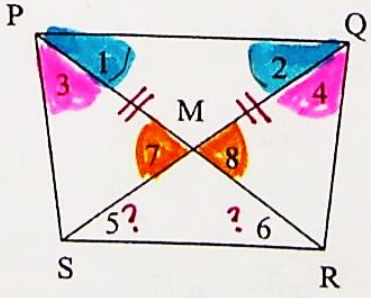
Prove: $\angle 1 \cong \angle 4$



Statements	Reasons
1) $\overline{XY} \cong \overline{XZ}$ (S)	1) Given
2) $\triangle XYZ \cong \triangle XZY$	2) If $\triangle$, then $\triangle$
3) $\overline{OY} \cong \overline{OZ}$	3) Given
4) $\triangle 2 \cong \triangle 3$	4) If $\triangle$, then $\triangle$
5) $\triangle 1 \cong \triangle 4$	5) Subtraction

3. Given: $\angle 1 \cong \angle 2$
$\angle 3 \cong \angle 4$

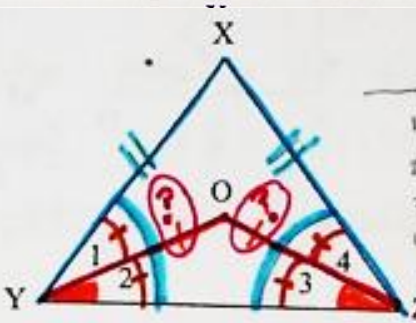
Prove: $\angle 5 \cong \angle 6$



Statements	Reasons
1) $\triangle 1 \cong \triangle 2$	1) Given
2) $\overline{PM} \cong \overline{QM}$ (S)	2) If $\triangle$, then $\triangle$
3) $\triangle 3 \cong \triangle 4$ (A)	3) Given
4) $\triangle 7 \cong \triangle 8$ (A)	4) Vertical $\triangle$s are $\cong$
5) $\triangle PMS \cong \triangle QMR$	5) ASA (3, 2, 4)
6) $\overline{SM} \cong \overline{RM}$	6) CPCTC
7) $\triangle 5 \cong \triangle 6$	7) If $\triangle$, then $\triangle$

4. Given: $\overline{XY} \cong \overline{XZ}$
$\overrightarrow{YO}$ bisects $\angle XYZ$
$\overrightarrow{ZO}$ bisects $\angle XZY$

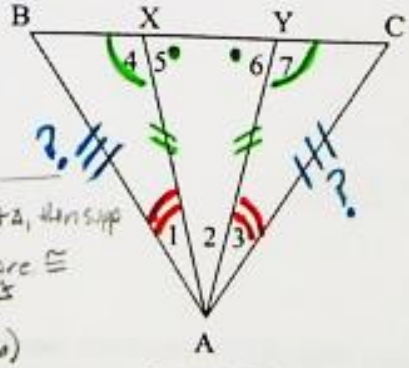
Prove: $\overline{YO} \cong \overline{ZO}$



Statements	Reasons
1) $\overline{XY} \cong \overline{XZ}$	1) Given
2) $\triangle XYZ \cong \triangle XZY$	2) If $\triangle$, then $\triangle$
3) $\overline{YO}$ bis $\angle XYZ$	3) Given
4) $\overline{ZO}$ bis $\angle XZY$	4) Given
5) $\angle 2 \cong \angle 3$	5) Division
6) $\overline{YO} \cong \overline{ZO}$	6) If $\triangle$, then $\triangle$

5. Given: $\angle 4 \cong \angle 7$
$\angle 1 \cong \angle 3$

Prove: Δ ABC is isosceles



Statements	Reasons
1) $\triangle 4 \cong \triangle 7$ (A)	1) Given
2) $\triangle 4$ supps $\triangle 5$	2) If 2 $\triangle$s form $\triangle$s, then supp
3) $\triangle 6$ supps $\triangle 7$	3) same as #2
4) $\triangle 5 \cong \triangle 6$	4) supps of $\cong \triangle$s are $\cong$
5) $\overline{AX} \cong \overline{AY}$ (S)	5) If $\triangle$, then $\triangle$
6) $\triangle 1 \cong \triangle 3$ (A)	6) Given
7) $\triangle BAX \cong \triangle CAY$	7) ASA (1, 5, 6)
8) $\overline{BA} \cong \overline{CA}$	8) CPCTC
9) $\triangle ABC$ is isosceles	9) If $\triangle$, then isosceles

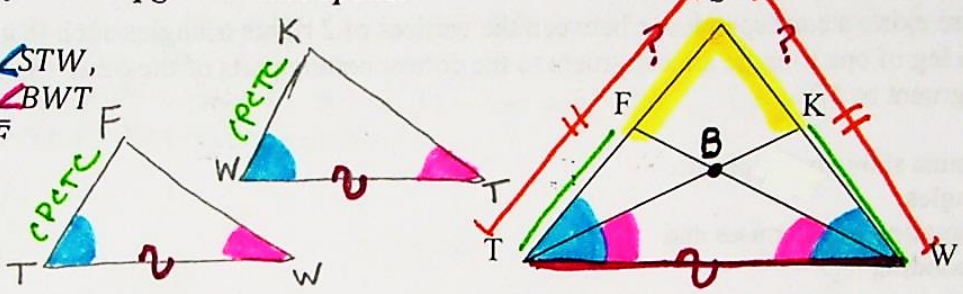
3.7 Angle-Side Theorems

Geometry Examples

Example 1) Proof from TE pg. 156 class opener

Given: $\angle SWT \cong \angle STW$, $\angle BTW \cong \angle BWT$

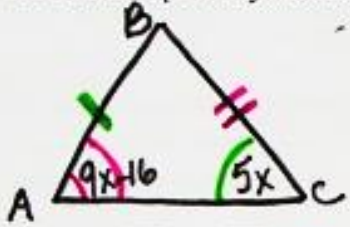
Prove: $\overline{SK} \cong \overline{SF}$



Statements	Reasons
1. $\triangle SWT \cong \triangle STW$ (A)	1. Given
2. $\triangle BTW \cong \triangle BWT$ (A)	2. Given
3. $\overline{TW} \cong \overline{TW}$ (S)	3. Reflexive
4. $\triangle FTW \cong \triangle KWT$	4. ASA (1, 3, 2)
5. $\overline{FT} \cong \overline{KW}$	5. CPCTC
6. $\overline{ST} \cong \overline{SW}$	6. If $\triangle$, then $\triangle$
7. $\overline{SK} \cong \overline{SF}$	7. Subtraction

1.) Given: ΔABC; $AB < BC$; $\angle A = 9x - 16$, $\angle C = 5x$

Write an inequality to describe the restrictions on x.



$9x - 16 > 5x$

$4x > 16$

$x > 4$

$9x - 16 + 5x < 180$

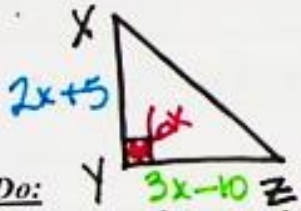
$14x < 196$

$x < 14$

$\boxed{4 < x < 14}$

2) Given: ΔXYZ; $\overline{XY} \perp \overline{YZ}$; $\angle Y = 6x$, $\overline{XY} = 2x + 5$, $\overline{YZ} = 3x - 10$.

Is ΔXYZ isosceles?



$6x = 90$

$x = 15$

$\overline{XY} = 2x+5 = 2(15)+5 = 30+5 = 35$

$\overline{YZ} = 3x-10 = 3(15)-10 = 45-10 = 35$

Yes!

You Do:

If you have not done so already, please read section 3.7, take notes & record/memorize any theorems.