3.6 Notes: "Types of Triangles"

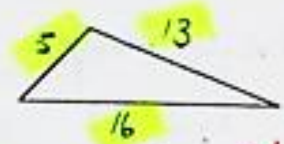
GEOMETRY 3.6
Types of Triangles

- There are 2 ways to classify triangles, by sides and by angles.
- For each definition below, rewrite in if-then form on the line below. Answer any questions.

Classify by sides

1. Scalene Δ – a triangle in which no 2 sides are congruent

Cond: If a triangle is scalene, then no 2 of its sides are congruent.



2. Isosceles Δ – a triangles in which at least 2 sides are congruent.

at minimum

Cond: If a triangle is isosceles, then at least 2 of its sides are congruent.

> The 2 congruent sides are called **legs**. The side which is not a leg is called **the base**. The angle that includes the legs is called the **vertex angle**.

3. Equilateral Δ – a triangle in which all 3 sides are congruent.

Cond: If a triangle is equilateral, then all 3 of its sides are congruent.

Question: According to these definitions, is a triangle that is equilateral also isosceles? Yes!

Question: Is a triangle that is isosceles also equilateral? NO!

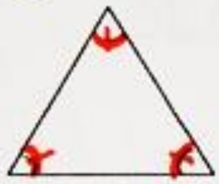
not necessarily, so NO!

Classify by angles:

Note: All triangles have at least two acute angles!

* Equiangular Δ – a triangle in which all 3 angles are congruent.

Cond.: If a triangle is equiangular, then all 3 angles are congruent

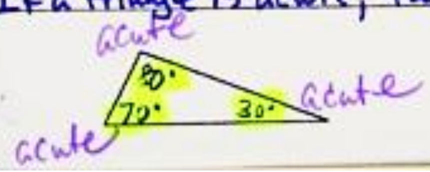


Question: How many degrees are in the 3 angles of any triangle? $\boxed{180^\circ}$

Question: How many degrees are in EACH angle of an equiangular triangle? $\frac{180}{3} = \boxed{60^\circ}$

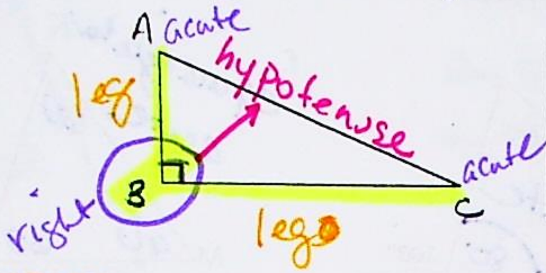
1. Acute Δ – a triangle in which all 3 angles are acute

Cond: If a triangle is acute, then all 3 angles are acute.



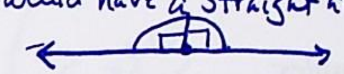
2. Right Δ – a triangle in which one of the angles is a right angle.

Cond: If a triangle is RIGHT, then one of the angles is a RIGHT angle.



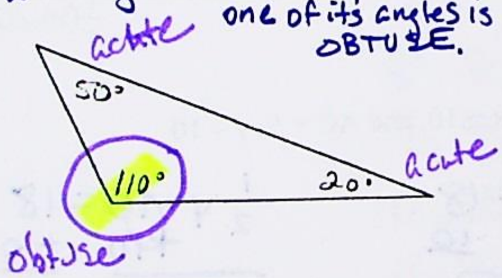
The sides which include the right angle are called **legs**. The side opposite the right angle is called the **hypotenuse**. The hypotenuse is always the longest side of any right triangle.

Question: Can a triangle have 2 right angles? Explain. No, the sum of all 3 angles is 180, Two Rt ∠'s = 180, so you wouldn't have a third ∠ etc.
* You would have a straight angle!



3. Obtuse Δ – a triangle in which one of the angles is obtuse.

Cond: If a triangle is obtuse, then one of its angles is OBTUSE.



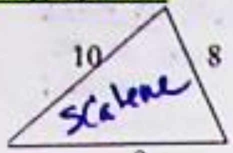
Question: Why, in the definition for an obtuse triangle, can there be only **one** obtuse angle? If 2 obtuse angles are added, then the sum will exceed the total for 3 angles in a Δ!

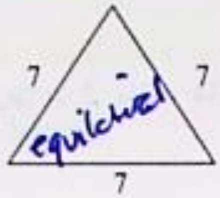
Ex: 91° + 91° = 182° no degrees left for a third ∠, exceeds a str ∠, too!

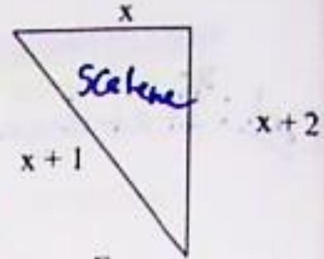
* See reason for 2 Rt ∠'s in a Δ

Go over pg. 145 (#2 and #3) together as a class.

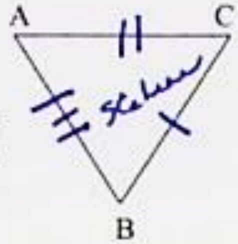
2) **Classify by SIDE**

a.


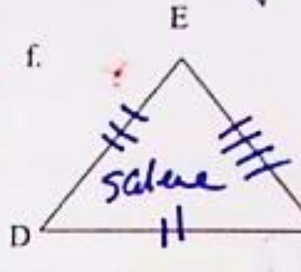
c.


e.


b.

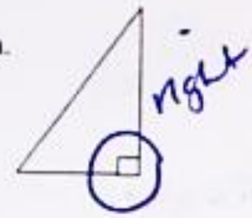

d.


AB > AC
AC > BC

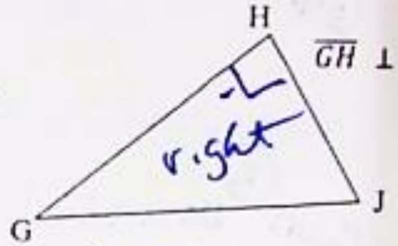
f.


DE > DF
EF > DE

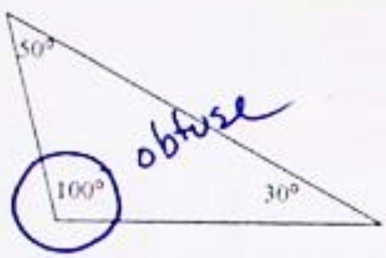
3) **Classify by ANGLE**

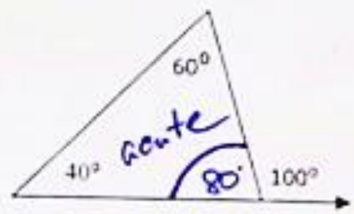
a.


c.


e.


$\overline{GH} \perp \overline{HJ}$

b.


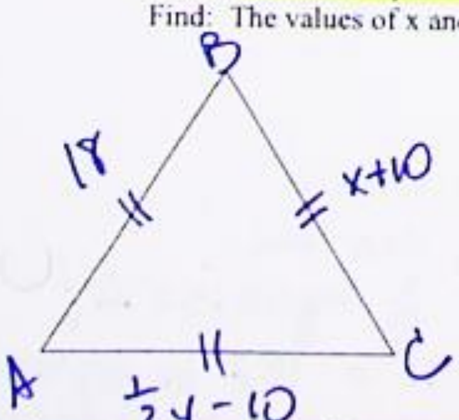
d.


f.


$\frac{1}{2}(m\angle K) = 30, \cdot 2$
$\frac{1}{3}(m\angle M) = 20, \cdot 3$
$\frac{1}{4}(m\angle O) = 15 \cdot 4$

Class Examples:

1.) Given: ΔABC is equilateral, AB = 18, BC = x + 10, and AC = ½ y − 10
Find: The values of x and y.



$$x + 10 = 18$$
$$-10 \quad -10$$
$$\boxed{x = 8}$$

$$\frac{1}{2}y - 10 = 18$$
$$+10 \quad +10$$
$$2 \cdot \frac{1}{2}y = 28 \cdot 2$$
$$\boxed{y = 56}$$

Example 2) The average of the lengths of the sides of $\triangle DEF$ is 20. If $DE = x + 7$, $EF = 3x - 4$, and $DF = 2x + 3$, how much longer than the average is the longest side?

$$\frac{DE+EF+DF}{3} = 20$$

$$3\left(\frac{6x+6}{\cancel{3}}\right) = 20(3)$$

$$6x + 6 = 60$$
$$\underline{-6 \quad\quad -6}$$

$$\frac{6x}{6} = \frac{54}{6}$$

$$\boxed{x = 9}$$

$$\begin{array}{r} 23 \\ -20 \\ \hline \end{array}$$

$\boxed{\text{3 units longer}}$

Example 3) How many different isosceles triangles can you find that have sides that are whole number lengths and that have a perimeter of 24?

Triangle Inequality Principle

~~1, 1, 22~~	$1+1 \not> 22$!
~~2, 2, 20~~	$2+2 \not> 20$!
~~3, 3, 18~~	$3+3 \not> 18$!
~~4, 4, 16~~	$4+4 \not> 16$!
~~5, 5, 14~~	$5+5 \not> 14$!
~~6, 6, 12~~	$6+6 \not> 12$!

Remember! The sum of any two sides must exceed the third side.

1. 7, 7, 10 ✓ — $7+7 > 10$ ✓
2. 8, 8, 8 ✓ — $8+8 > 8$ ✓
3. 9, 9, 6 ✓ — $9+9 > 6$ ✓
4. 10, 10, 4 ✓ — $10+10 > 4$ ✓
5. 11, 11, 2 ✓ — $11+11 > 2$ ✓

~~12, 12, 0~~

5 Δ's

Yes! By definition of "isosceles" this equilateral Δ is one, too!

$\boxed{\text{We found 5 isosceles triangles}}$