2.7 Transitive and Substitution Properties

Objectives:

1. *Apply the transitive properties of angles and segments*
2. *Apply the Substitution Property*

- In algebra you learned that the transitive property looked like this:

  If a = b and b = c, then a = c

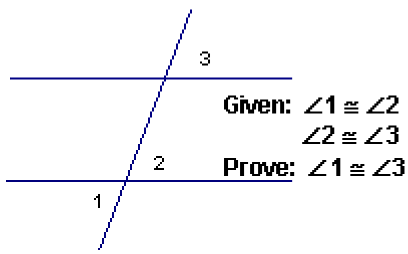
- In chapter 1, section 8 you learned "chains of reasoning" that followed this form:

  If $p \Rightarrow q$ and $q \Rightarrow r$, then $p \Rightarrow r$

- In geometry we use the ***Transitive Property*** when we have a series of congruent segments or angles that we wish to make or prove congruent.

  If $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 3$, then $\angle 1 \cong \angle 3$

Example:

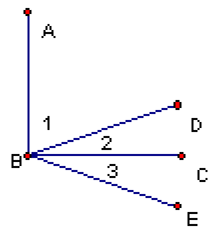


1. $\angle 1 \cong \angle 2$	1. Given
2. $\angle 2 \cong \angle 3$	2. Given
3. $\angle 1 \cong \angle 3$	2. Transitive Property

Don't you agree that this property looks very much like the chain rule $a \Rightarrow b,\ b \Rightarrow c,\ \text{then } a \Rightarrow c$? The difference is that you are using congruence or = signs and instead of the first *causing or implying ($\Rightarrow$)* the last, the first has to be = or congruent to the last.

Substitution means you are replacing a quantity or figure with another quantity or figure that the original is congruent or equal to. So, ***transitive*** *<u>is</u> a form of* ***substitution***. However, *substitution* has the added ability of replacing only one part of an equation or expression with an equal quantity.

Example:



Given: $\angle 1$ is complementary to $\angle 3$

$\overrightarrow{BC}$ bisects $\angle DBE$

Prove: $\angle ABC$ is a right angle

1. $\angle 1$ is complementary to $\angle 3$	1. Given
2. $\overrightarrow{BC}$ bisects $\angle DBE$	2. Given
3. $\angle 2 \cong \angle 3$	3. Def. of an angle bisector
4. $\angle 1$ is complementary to $\angle 2$	4. Substitution
5. $\angle 1 + \angle 2 = 90°$	5. Definition of complementary
6. $\angle 1 + \angle 2 = \angle ABC$	6. Angle Addition
7. $\angle ABC = 90°$	7. Transitive Property
8. $\angle ABC$ is a right angle	8. Definition of a right angle

Note: Notice that this proof uses ***both*** the substitution and transitive properties. When you use transitive there is a flow from one to the next as they are both equal to the same quantity, but in substitution you are only replacing part of the side of the equation i.e. angle 3 is replaced by angle 2. If you were to write substitution for the reason for number 7, it is acceptable as you have replaced a quantity with something it is equal to.

In other words:

Substitution works for transitive, but transitive does not *always* work for substitution.

Theorem 16: If angles (or segments) are congruent to the same angle (or segment), they are congruent to each other. *(Transitive Property)*

Theorem 17: If angles (or segments) are congruent to congruent angles (or segments), they are congruent to each other. *(Transitive Property)*