## Properties of Equality and Congruence

## John is taller than Kevin and Kevin is taller than Louis.

How do the heights of John and Louis compare?
We can analyze the situation with the aid of a simple diagram. The diagram leads us to conclude that John must be taller than Louis.
height


Using the mathematical symbol for greater than $(>)$, the height relationships can be represented by the following series of inequality statements:

$$
\begin{gathered}
\text { If } J>K \\
\text { and } K>L \\
\text { then, } J>L
\end{gathered}
$$

Without directly comparing John with Louis, we have used the transitive property to conclude that John's height is greater than Louis' height. The "greater than" relation is an example of a relation that possesses the transitive property.

Is friendship a transitive relation? If Ashton is Jordan's friend, and Jordan is Stephanie's friend, does that mean that Ashton must also be Stephanie's friend? Actually, no! Some relations possess the transitive property, white others do not.

The equality $(=)$ and congruence $(\underset{=}{( })$ relations possess the transitive property.

For example, if angle $A$ is congruent to angle $B$, and angle $B$ is congruent to angle $C$, then angle $A$ must be congruent to angle $C$. Another way of looking at this interrelationship between angles $A, B$, and $C$ is that angles $A$ and $C$ are each congruent to angle B and must, therefore, be congruent to each other.


| Transitive Property | Equality Example |  | Congruence Example |  |
| :--- | :--- | :--- | :--- | :--- |
| If two quantities are equal | If | $A B=C D$ | If | $\angle X \cong \angle Y$ |
| (or congruent) to the same quantity, | and | $C D=E F$ | and | $\angle Y \cong \angle Z$ |
| then they are equal (or congruent) to |  |  |  |  |
| each other. | then | $A B=E F$ | then | $\angle X \cong \angle Z$ |

Another useful property of the equality relation is the SUBSTITUTION Property of Equality.

| Substitution Property | Equality Example | Congruence Example |
| :---: | :---: | :---: |
| This is when an equivalent amount (or measure) may replace another expression in an equation (or congruence situation) | $\text { If } A B=3+4 \Leftrightarrow A B=7$ <br> We may substitute 7 in place of the numerical expression $(3+4)$ on the right side of the original equation. | Given: $m \angle 1+m \angle 2=90$ $m \angle 2=m \angle 3 .$ <br> Concl: $m \angle 1+m \angle 3=90$ <br> Reason: Substitution Property The $m \angle 3$ replaces its equal, $m \angle 2$ in the first equation stated in the given. |

Now we will examine situations where either the transitive or substitution properties are used to justify conclusions.

## Example \#1

Given: $\angle 1 \cong \angle 2$
$\angle 2 \cong \angle 3$
Concl: $\angle 1 \cong \angle 3$

Reason: ?


Solution: Since both angles 1 and 3 are congruent to the same angle, angle 2 , they must be congruent to each other. This is the Transitive Property of congruence. Since we may only substitute equals in equations, we do NOT have a substitution property of congruence.

### 2.7 Transitive and Substitution Properties of Equality

## Example \#2

Given: $m \angle 1 \cong m \angle 4$,

$$
\begin{aligned}
& m \angle 3 \cong m \angle 5 \\
& m \angle 4+m \angle 2+m \angle 5=180
\end{aligned}
$$

Conclusion: $m \angle 1+m \angle 2+m \angle 3=180$


Reason: ?

Solution: Substitution Property. In the equation stated lastly in the givens, the measures of angles 4 and 5 are replaced by their equals, the measures of angles 1 and 3 , respectively.

## Example \#3

Given: RS = SM (equation 1) TW = SM (equation 2 )
Conclusion: RS = TW


Reason: ?

Solution: Since RS and TW are both equal to the same quantity, SM, they must be equal to each other.
This is the Transitive Property.
~or~
In equation (1), SM may be replaced by its equal, TW. We are using the information in equation (2) to make a substitution in equation (1). Hence, the conclusion can also be justified by using the Substitution Property.

## Example \#4

Given: $C$ is the midpoint of $\overline{A D}$

$$
\mathrm{AC}=\mathrm{CE}
$$

Conclusion: $\mathrm{CD}=\mathrm{CE}$
Reason: ?


Solution: $A C=C D$ since point $C$ is the midpoint of $\overline{A D}$. E

We now have the set of relationships:

$$
\begin{array}{cc}
A C=C D & (\text { equation } 1) \\
A C=C E & (\text { equation } 2)
\end{array}
$$

Since CD and CE are both equal to the same quantity (AC) they must be equal to each other. Therefore, $C D=C E$ by the transitive property of equality.

I hope these examples have clarified for you how the transitive and substitution properties of equality, in certain situations, may be used interchangeably (see examples 3 and 4).

In examples $3 \& 4$, there are two equations that state that two quantities are each equal to the same quantity, thus leading to either substitution or transitive properties of equality. ©

