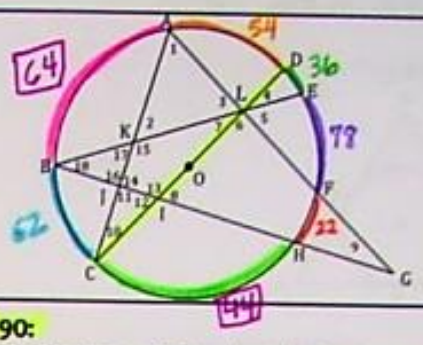
CHAPTER 10.6 MORE ANGLE-ARC THEOREMS Name:

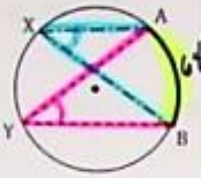
GETTING STARTED! Given: ⊙O, $\overset{\frown}{AD} = 54$, $\overset{\frown}{BC} = 62$, $\overset{\frown}{DE} = 36$, $\overset{\frown}{FH} = 22$, $\overset{\frown}{EF} = 78$.

Find the measures of the missing arcs and numbered angles.

$\overset{\frown}{CH}$ = 44	∠3 = 71	∠7 = 49	∠11 = 54	∠15 = 104
$\overset{\frown}{AB}$ = 64	∠4 = 49	∠8 = 99	∠12 = 99	∠16 = 54
∠1 = 33	∠5 = 71	∠9 = 21	∠13 = 81	∠17 = 76
∠2 = 76	∠6 = 60	∠10 = 27	∠14 = 126	∠18 = 50



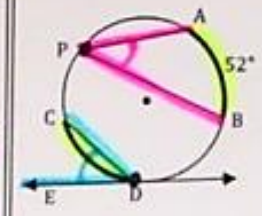
THEOREM 89:
If two inscribed or tangent-chord angles intercept the same arc, then they are congruent.



What conclusion can be drawn in the diagram and why?
(same intercepted arc) ∠X ≅ ∠Y

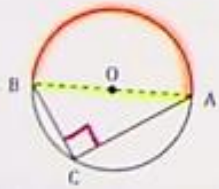
If m$\overset{\frown}{AB}$ = 64, then find:
a) m∠X = 32° ½(64)
b) m∠Y = 32° ½(64)

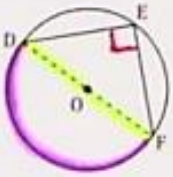
THEOREM 90:
If two inscribed or tangent-chord angles intercept congruent arcs, then they are congruent.



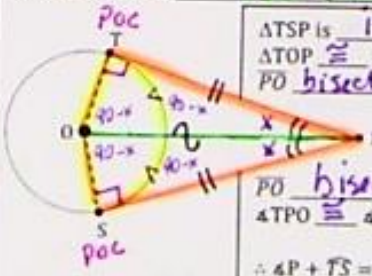
Given: $\overset{\frown}{AB} \cong \overset{\frown}{CD} = 52°$
(intercept ≅ arcs)
∠APB ≅ ∠CDE
Find:
a) m∠P = 26° ½(52)
b) m∠EDC = 26° ½(52)

THEOREM 91:
An angle inscribed on a semi-circle is a RIGHT angle (90°).



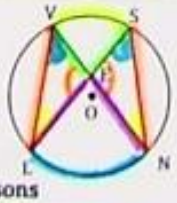


THEOREM 92:
The sum of the measures of a tangent-tangent angle and its minor arc is 180° (they are supp.)



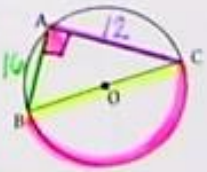
ΔTSP is isosceles
ΔTOP ≅ ΔSOP by SSS
$\overline{PO}$ bisects central ∠ TOS
$\overline{PO}$ bisects $\overline{TS}$
∠TPO ≅ ∠SPO by CPCTC
∴ m∠P + $\overset{\frown}{TS}$ = 180°

Example 1:
Given: ⊙O
Conclusion: ΔLVE ~ ΔNSE, EV · EN = EL · SE



Statements	Reasons
1. ⊙O	1. Given
2. ∠V ≅ ∠S (A)	2. ∠'s inscribed on same arc are ≅
3. ∠L ≅ ∠N (A)	3. Same as #2
4. ΔLVE ~ ΔNSE	4. AA~ (2, 3)
5. $\frac{EV}{SE} = \frac{EL}{EN}$	5. CSSTP
6. EV · EN = EL · SE	6. Means-Extremes Products Thm

Example 2: In ⊙O, $\overline{BC}$ is a diameter, AC = 12 mm, and BA = 16 mm.
Find the radius of the circle.



$\frac{12}{4}$ $\frac{16}{4}$ $\frac{x}{4}$
3 - 4 - 5 Family!
$\frac{x}{4} = 5$ ∴ x = 20 (BC)
r = ½(BC) = ½(20) = 10 mm

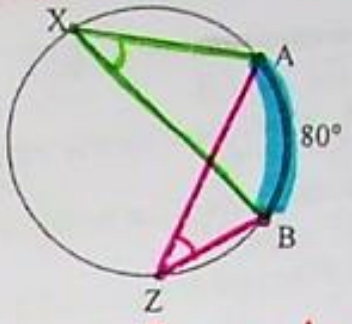
10.6: More Angle-Arc Theorems

Theorem: If two inscribed or tangent-chord angles intercept the **same** arc, then they are congruent.

- Consider a circle with $m\overset{\frown}{AB} = 80$
- $\angle X$ and $\angle Z$ are *inscribed* angles since their vertices lie **ON** the circle and their sides are chords.

1. What is $m\angle X$? 40°
2. What is $m\angle Z$? 40°

$\frac{1}{2}(80)$

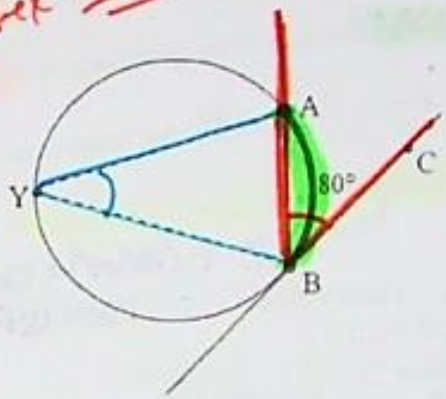


Vertex ON ⊙ ∴ $m\angle = \frac{1}{2}$ arc

- Consider a circle with $m\overset{\frown}{AB} = 80$
- $\angle Y$ in an *inscribed* angle; $\angle ABC$ is a tangent-chord angle

3. What is $m\angle Y$? 40°
4. What is $m\angle ABC$? 40°

$\frac{1}{2}(80)$

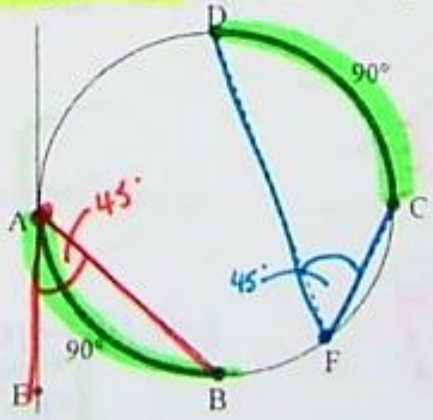


Theorem: If two inscribed or tangent-chord angles intercept **congruent** arcs, then they are congruent.

- Consider a circle with $m\overset{\frown}{AB} = 90$ and $m\overset{\frown}{CD} = 90$
- $\angle F$ in an *inscribed* angle; $\angle BAE$ is a tangent-chord angle

5. What is $m\angle F$? 45°
6. What is $m\angle BAE$? 45°

$\frac{1}{2}(90)$

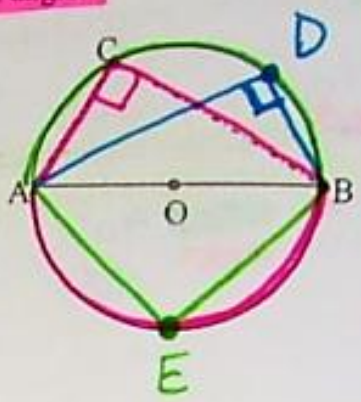


Vertex "ON"

$m\angle = \frac{1}{2}$ arc!

10.6 (Cont'd)

Theorem: If an angle is inscribed in a semi-circle, then it is a right angle.



- Consider ⊙O and diameter $\overline{AB}$
  7. What is $m\angle C$? (inscribed) 90
- Draw another point (D) on $\overset{\frown}{ACB}$.
- Draw $\overline{DA}$ and $\overline{DB}$
  8. What kind of angle is $\angle D$? Right Δ
- Draw another point (E) on the *lower* semi-circle.
- Draw $\overline{EA}$ and $\overline{EB}$
  9. What kind of angle is $\angle E$? Right Δ

Theorem: The sum of the measures of a tangent-tangent angle and its minor arc is 180.

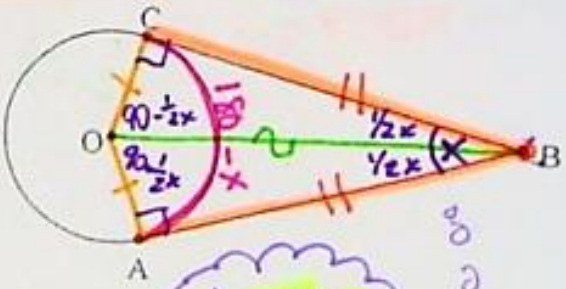
+ =

Given: $\overline{BA}$ and $\overline{BC}$ are tangent to ⊙O

Conclusion: $m\angle B + m\overset{\frown}{AC} = 180$

$x + (180 - x) = 180$

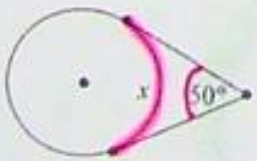
$180 = 180$ ✓



Proof:
- ✓ Let $m\angle B = x$
- ✓ Draw $\overline{OC}$, $\overline{OA}$, and $\overline{OB}$
- ✓ What is $m\angle OCB$? $m\angle OAB$? 90°
- ✓ Why is $\Delta OCB \cong \Delta OAB$? HL or SAS or SSS
- ✓ What is $m\angle OBC$? $m\angle OBA$?
- ✓ What is $m\angle BOC$? $m\angle BOA$? $90 - \frac{1}{2}x$ and $90 - \frac{1}{2}x$
- What is $m\overset{\frown}{AC}$? $(90 - \frac{1}{2}x) + (90 - \frac{1}{2}x) = 180 - x$

they are SUPP (the angle & the arc)

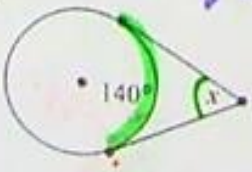
10.



$x = 180 - 50$

130°

11.



$x = 180 - 140$

40°

12. 360 − 250

180 − 110

70°