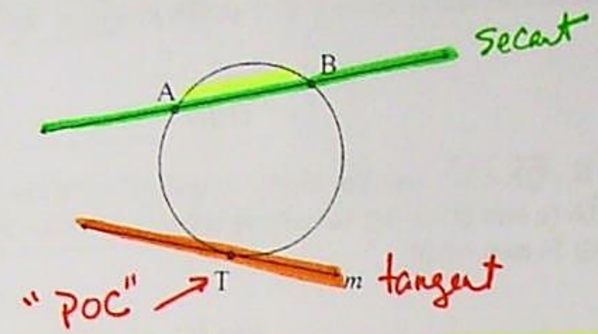
10.4 Secants & Tangents

Worksheet: 10.4

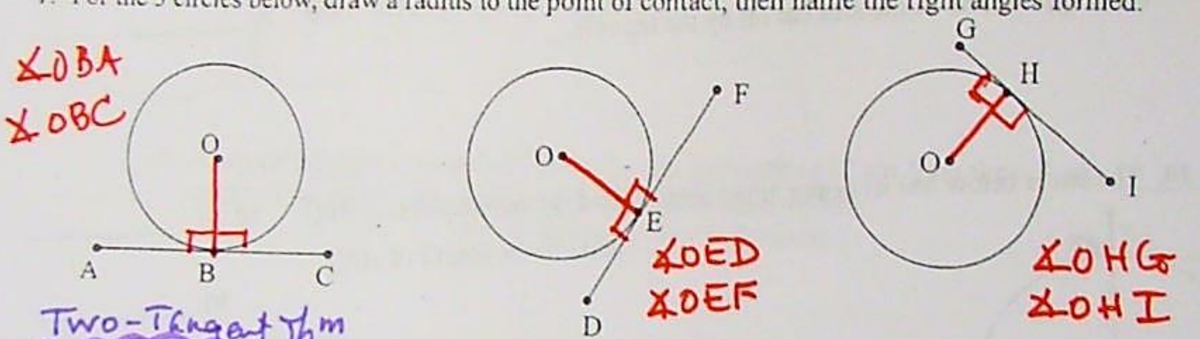
Defn.: a **secant** is a line that intersects a circle at exactly 2 points. (*note*: every **secant** contains a chord)

Defn.: a **tangent** is a line that intersects a circle at exactly 1 point, called the *point of tangency* or the *point of contact*.



Postulate: If a radius is drawn to the point of contact, then it is perpendicular to the tangent.

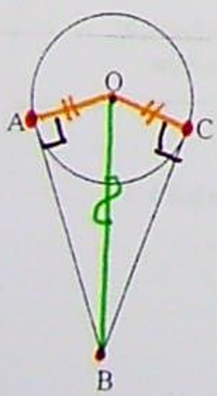
1. For the 3 circles below, draw a radius to the point of contact, then name the right angles formed.



Two-Tangent Thm

Ice-cream cone Theorem: if 2 tangent segments are drawn to a circle from a common exterior point, then they are ≅.

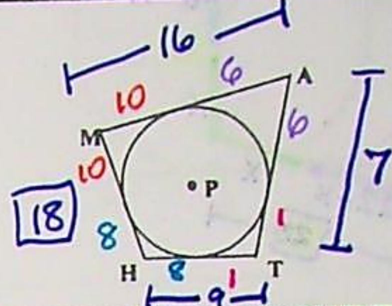
Given: ⊙O; $\overline{BA}$ and $\overline{BC}$ are tangent segments.
Prove: $\overline{BA} \cong \overline{BC}$



	Statements	Reasons
	1. ⊙O	1. Given
	2. $\overline{BA}$ and $\overline{BC}$ are tangent segments	2. Given
	3. draw $\overline{OA}, \overline{OB}, \overline{OC}$	3. 2 pts det a seg
L	4. $\overline{OA} \cong \overline{OC}$	4. all radii of a ⊙ are ≅
H	5. $\overline{OB} \cong \overline{OB}$	5. reflexive property
	6. $\overline{OA} \perp \overline{AB}$; $\overline{OC} \perp \overline{BC}$	6. A radius drawn to POC of tangent is ⊥ to tangent
*	7. ∠OAB and ∠OCB are right ∠s	7. ⊥ segs form Rt ∡'s
	8. △OAB ≅ △OCB	8. HL (7; 5, 4)
	9. $\overline{BA} \cong \overline{BC}$	9. CPCTC

The Famous "Walk-Around" Problem

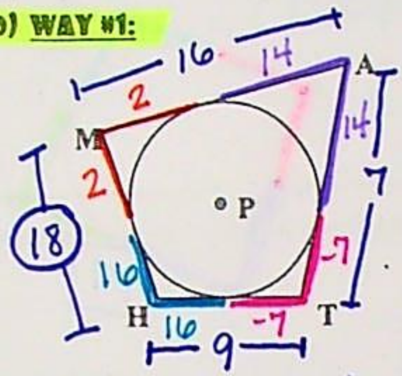
Problem: $\odot P$ is tangent to each side of MATH. MA = 16, AT = 7, TH = 9. Find MH.



THERE ARE 3 INCREDIBLE WAYS, OF VARYING RIDONKULOUSNESS, TO SOLVE THIS!!!

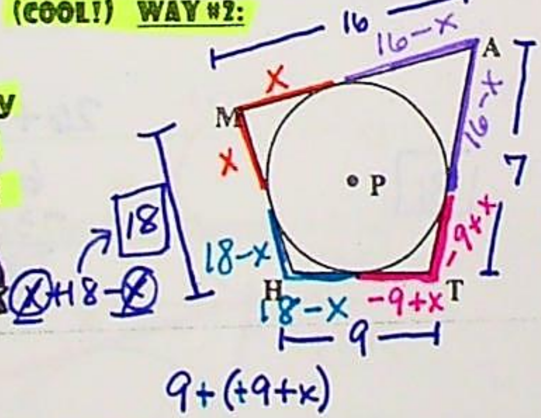
Semi-Sick

(PRETTY GOOD) WAY #1:



(COOL!) WAY #2:

Pretty Darn Sick!

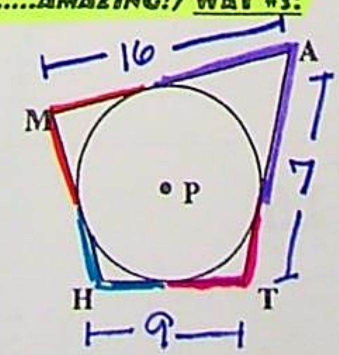


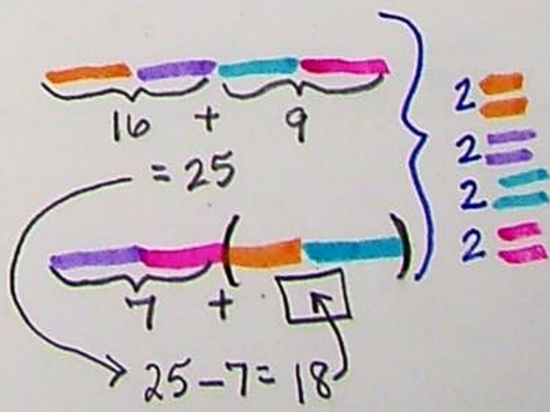
$7-(16-x)$

$\boxed{-9+x}$

(WHOA-OAH.....AMAZING!) WAY #3:

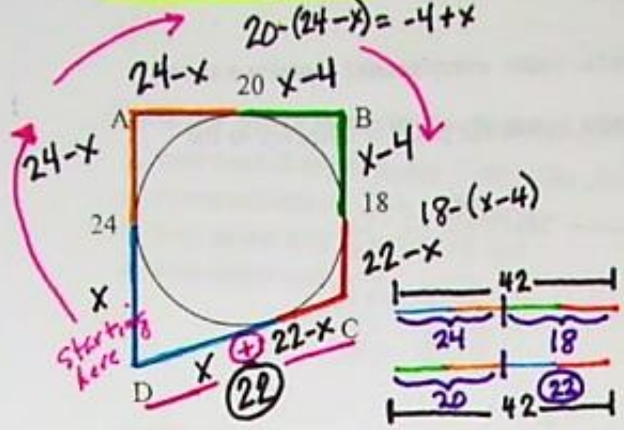
Ridonkulously Sick!!

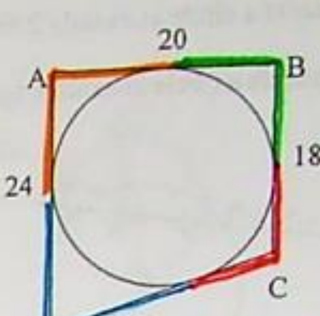




10.4 Application of the "Two-Tangent" Theorem!

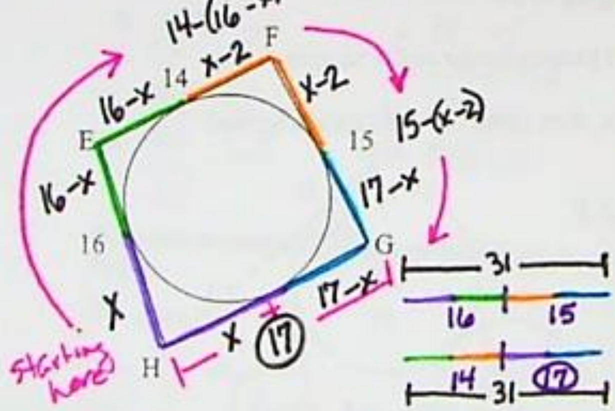
2. **Walk-around problems.** Find the missing side of each quadrilateral.

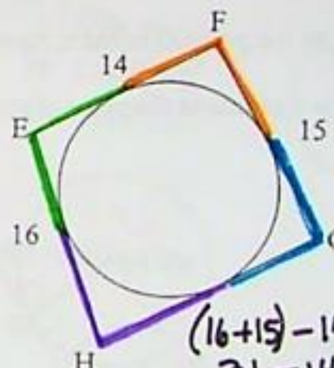




$(24+18)-20=\overline{DC}$

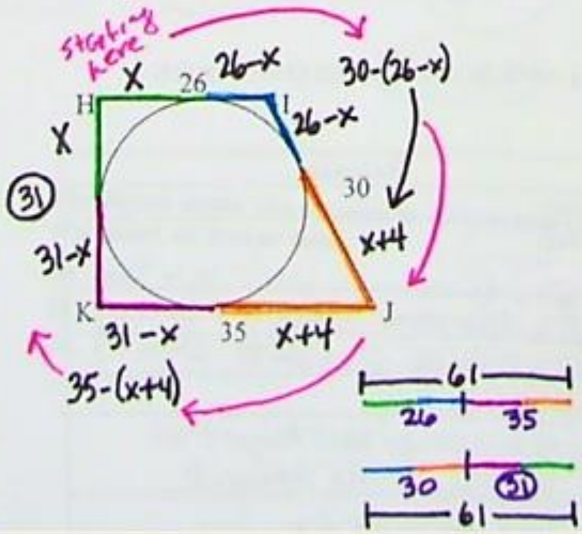
$42-20=22$

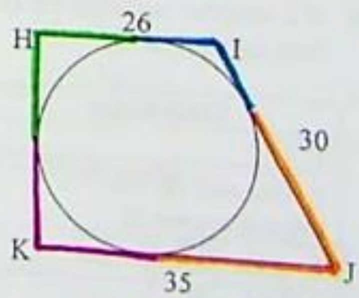




$(16+15)-14=\overline{HG}$

$31-14=17$





$(26+35)-30=\overline{HK}$

$61-30=31$

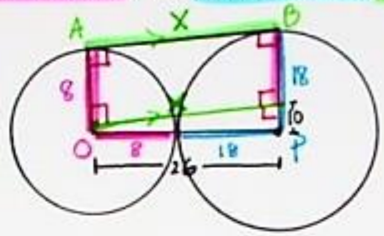
10.4- Common Tangent Procedure

1. Draw an appropriate diagram.
2. Draw the segment connecting their centers.
3. Draw the radii to the points of contact.
4. Through the center of the smaller circle, draw a line parallel to the common tangent.
5. Extend any radius if necessary to obtain right triangles and rectangle.
6. Use the Pythagorean Theorem and properties of a rectangle.

Sample problems. Draw a clear diagram for each problem and solve.

1. Circles O and P are tangent to each other and have a common *external* tangent $\overline{AB}$. If the radius of ⊙O is 8 and the radius of ⊙P is 18, find the length of the common tangent.

$\overline{AB} = 24$



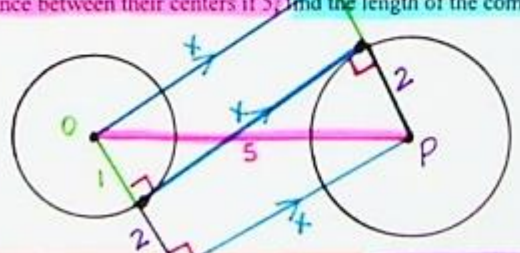
$18 - 8 = 10$

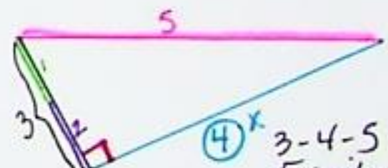
$\frac{10}{2}$ $\boxed{\frac{x}{2}}$ $\frac{26}{2}$

5 - 12 - 13 Family!

$\frac{x}{2} = 12$ $\boxed{\therefore x = 24}$

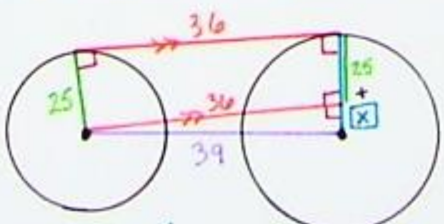
2. Circles O and P have a common *internal* tangent. The radius of ⊙O is 1 and the radius of ⊙P is 2. If the distance between their centers if 5, find the length of the common tangent.

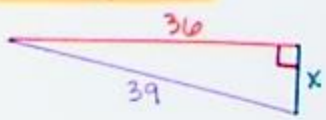




(4) x 3-4-5 Family!

3. Circles O and P have a common *external* tangent. Their centers are 39 cm apart. If the radius of the smaller circle is 25 and the length of the common tangent is 36, find the radius of the larger circle.





$\boxed{\frac{x}{3}}$ $\frac{36}{3}$ $\frac{39}{3}$

5 - 12 - 13 Family!

$\frac{x}{3} = 5$ $\therefore \boxed{x = 15}$

$r = 25 + 15$

$\boxed{r = 40\text{cm}}$